



Survival Analysis

Module I: A Bird's Eye View

Dr. Mark Williamson

DaCCoTA

University of North Dakota

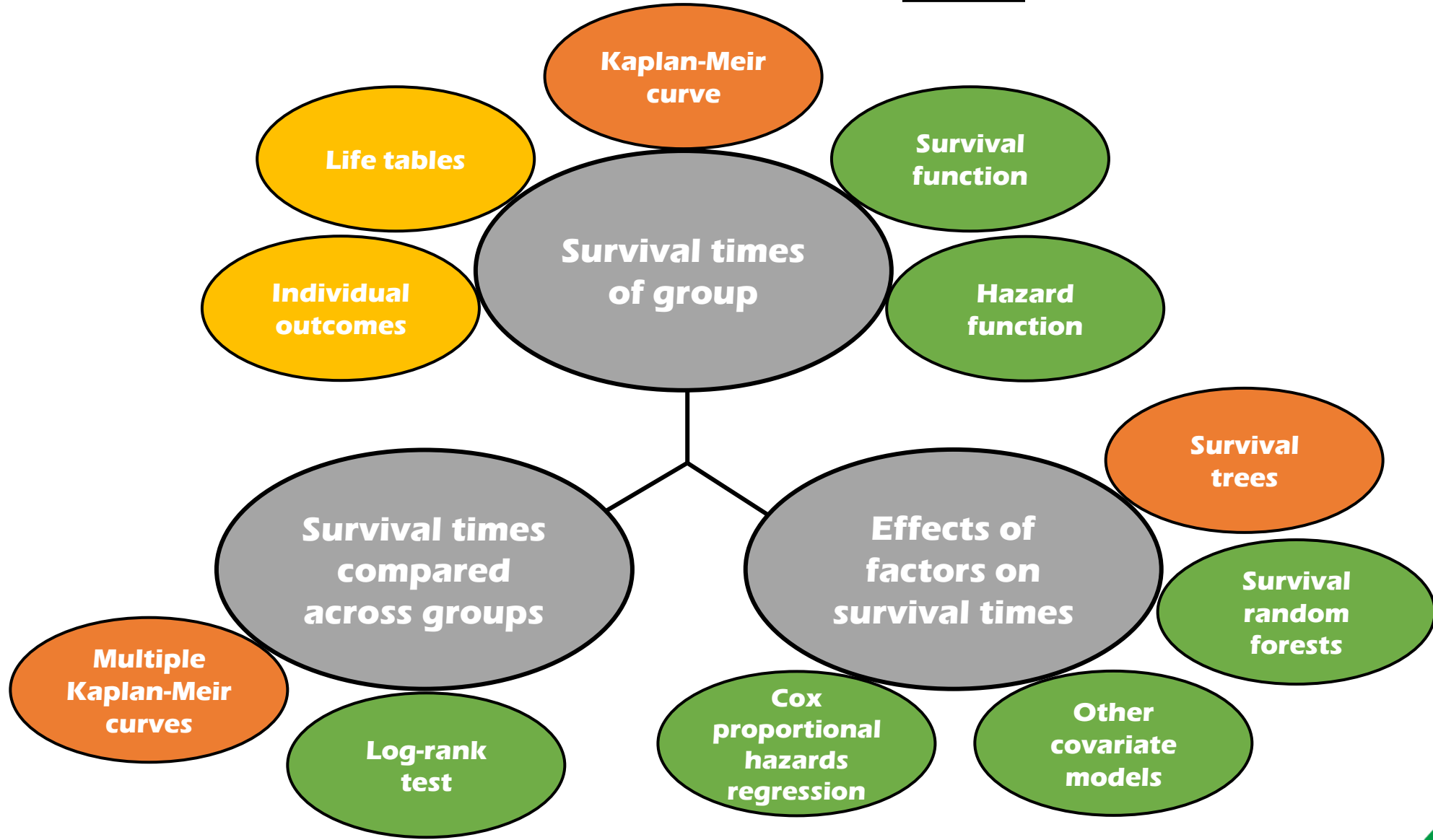
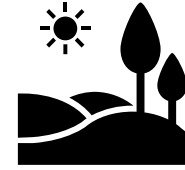
Introduction



What is Survival Analysis?

- Fundamentally time-to-event data [1,2]
 - Cohort is measured over time for event
 - Event may be death/disease/failure/bankruptcy/etc.
 - Important in fields such as epidemiology, engineering, economics, etc.
- Survival analysis is used in three typical ways:
 - Survival time of a group
 - Survival time across groups
 - Survival time based on factors
- Typical approaches are visualization through survival curves and modeling using methods such as the log-rank test or Cox regression
- Important terms include **event**, **time**, **censoring**, **survival function**, and **hazard function** [1]
- Basic rules of survival [3]
 - “No one starts out dead” -> $S(t)=1 @ t=0$
 - “Everyone dies eventually” -> $S(t)=0 @ t=\infty$
 - “Once you are dead, you stay dead” -> $S(t)$ never increases

Landscape



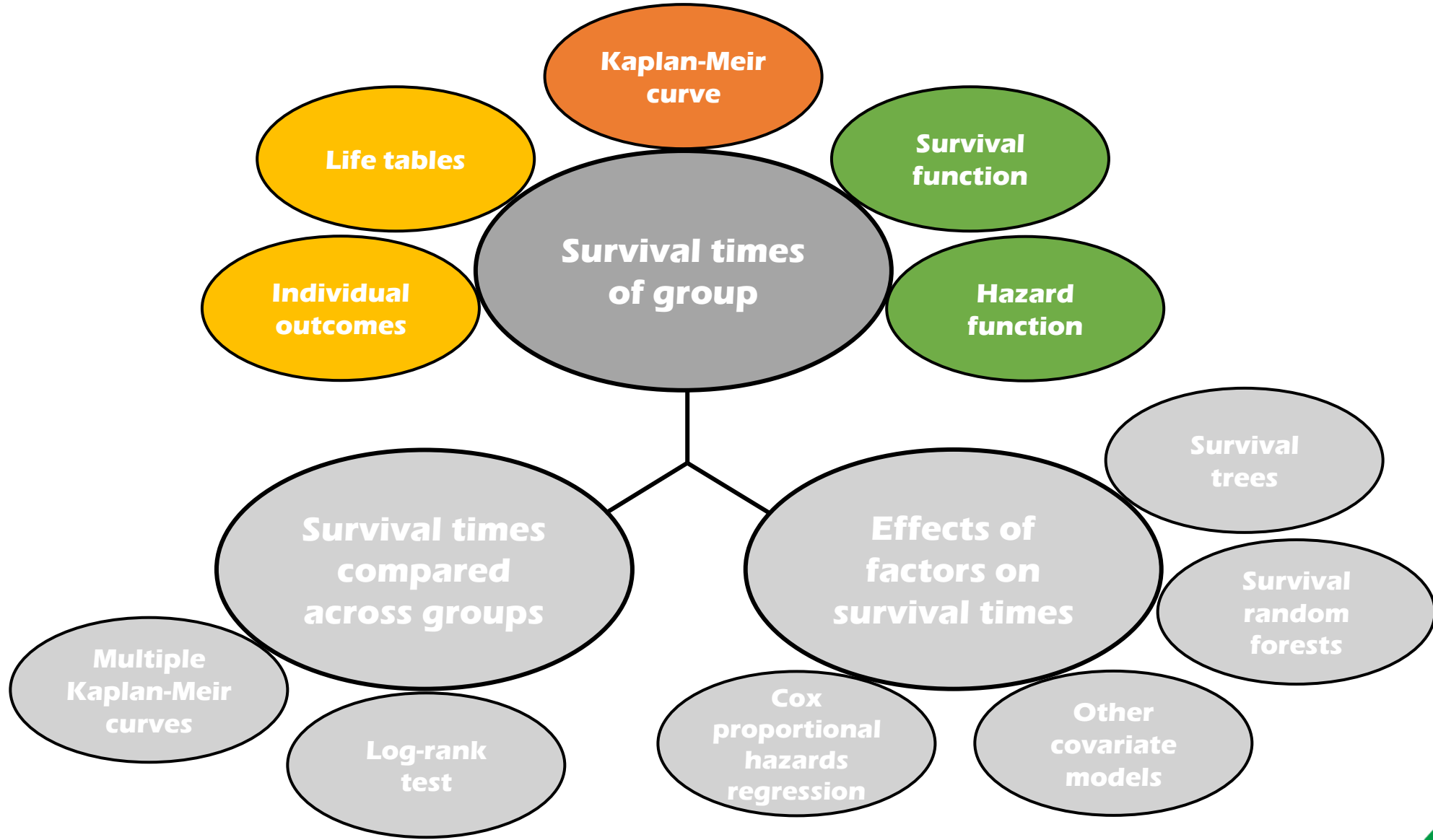
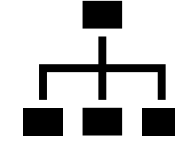
Concepts

Models/
Functions

Graphs

Other
visuals

Structure and Uses



Structure and Uses 2

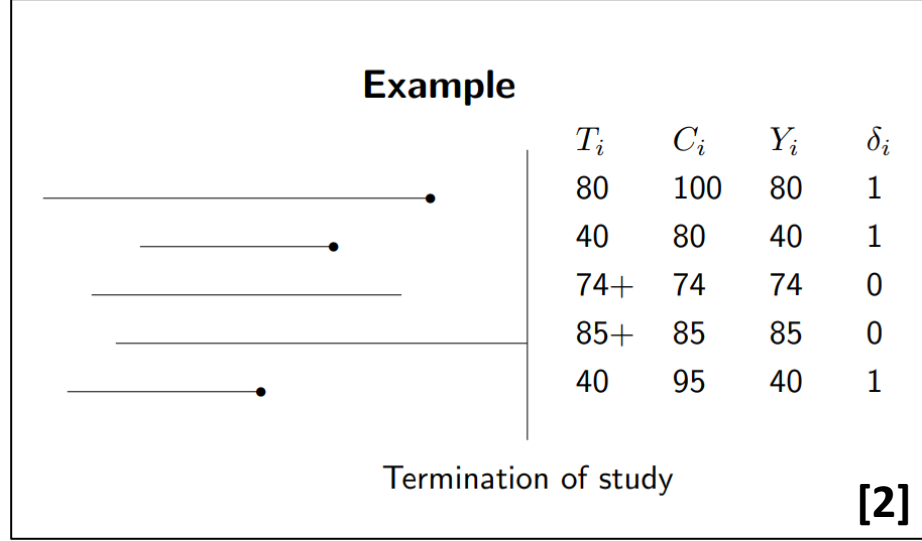
Survival times of group

- Individual outcomes
- Life tables

Kaplan-Meier curve

Survival function

Hazard function



A	B	C	D	E	F	G	H
Yrs in Trial	Died		t	d	n	1-d/n	S(t)
3	1		0		18		1
5	1		2	1	18	0.944444	0.944444
8	1		3	1	17	0.941176	0.888889
10	0		5	3	16	0.8125	0.722222
5	1		8	3	12	0.75	0.541667
5	0		9	0	9		0.541667
8	1		10	0	8		0.541667
12	1		11	2	6	0.666667	0.361111
15	0		12	2	4	0.5	0.180556

[5]

H	I	J	K	L	M	N	O	P	Q	R	S
x	nx	bx	mx	lx	lx mx	lx mx x			coeff.	decription	equation
0	30	0	0	1	0	0			x	number of days	
1	30	0	0	1	0	0			nx	Number of living individuals	
2	30	18	0.6	1	0	0			bx	number of births	
3	29	425	14.65517	0.966667	0.6	1.8			mx	number of births/number of survivors	bx/nx
4	29	340	11.72414	0.966667	14.16667	56.66667			lx	survivors	nx/N (total number of ind)
5	29	0	0	0.966667	11.33333	56.66667			dx	deaths	
6	27	325	12.03704	0.9	0	0			lxmx		
7	26	493	18.96154	0.866667	10.83333	75.83333					
8	24	217	9.041667	0.8	16.43333	131.4667			Ro	net reproductive rate	SUM(lxmx)*
9	23	130	5.652174	0.766667	7.233333	65.1			G	generation time	SUM(lx*mx*x)/SUM(lx*mx)
10	15	126	8.4	0.5	4.333333	43.33333			r	intrinsic rate of increase	ln(Ro)/G
11	15	0	0	0.5	4.2	46.2					
12	15	0	0	0.5	0	0					
13	15	3	0.2	0.5	0	0					
14	8	3	0.375	0.266667	0.1	1.4					
15	8	1	0.125	0.266667	0.1	1.5					
16	7	0	0	0.233333	0.033333	0.533333					
17	0	0	0	0	0	0					
18	0	0	0	0	0	0					

Structure and Uses 2

Survival times of group

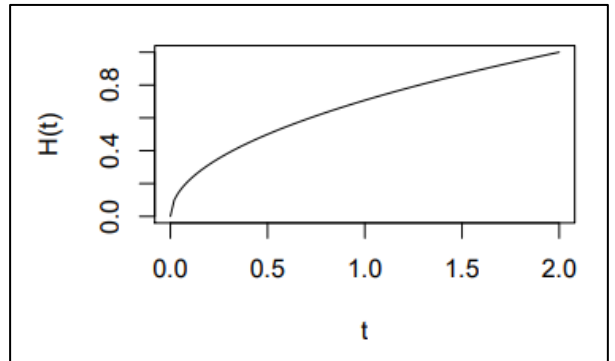
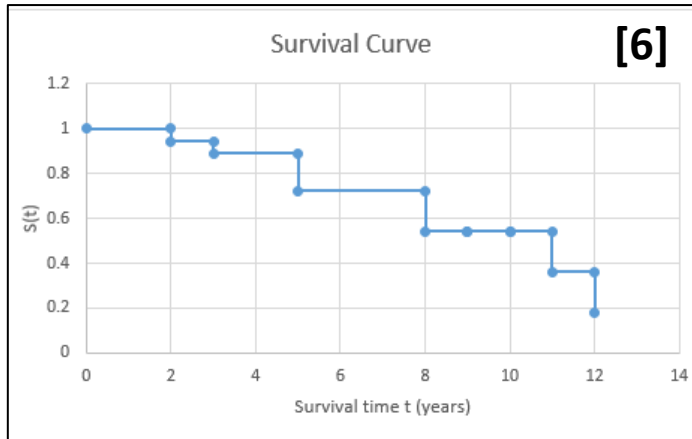
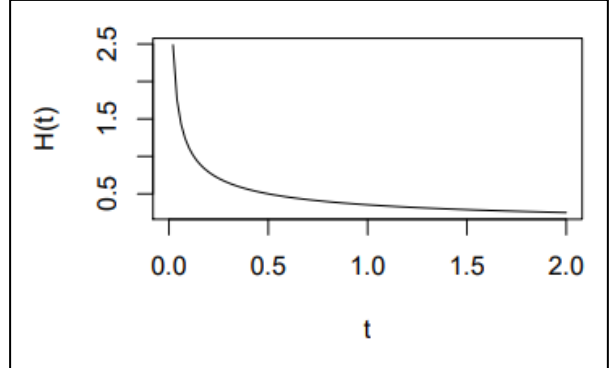
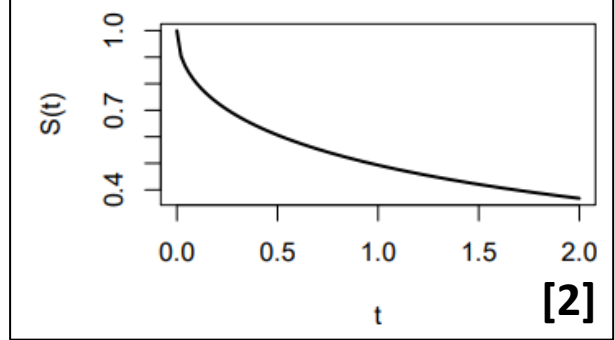
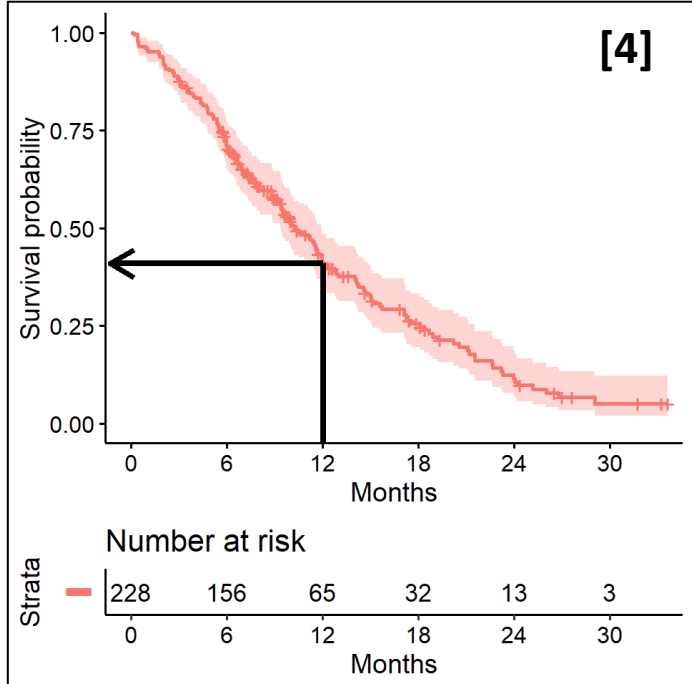
Individual outcomes

Life tables

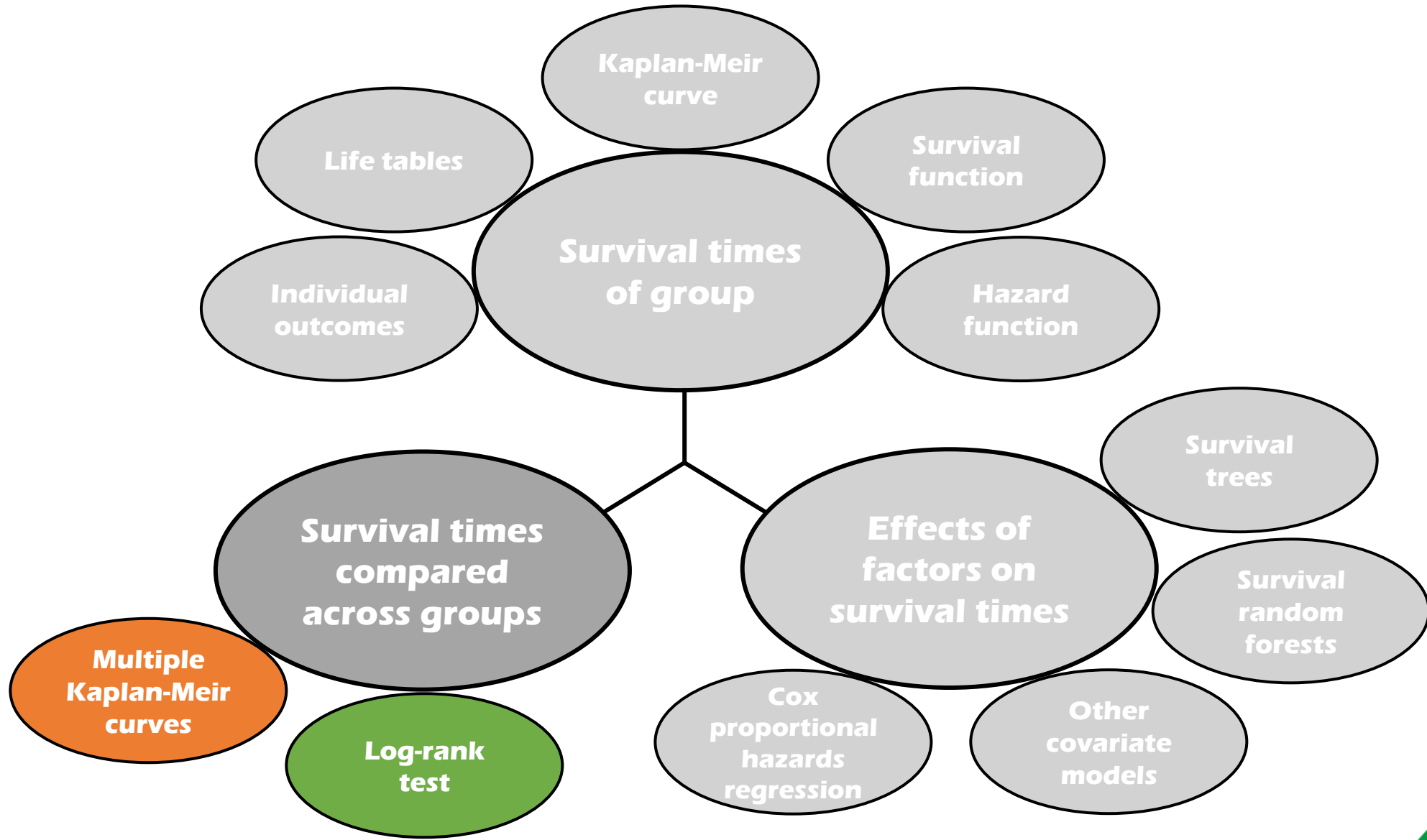
Kaplan-Meier curve

Survival function

Hazard function



Structure and Uses 3

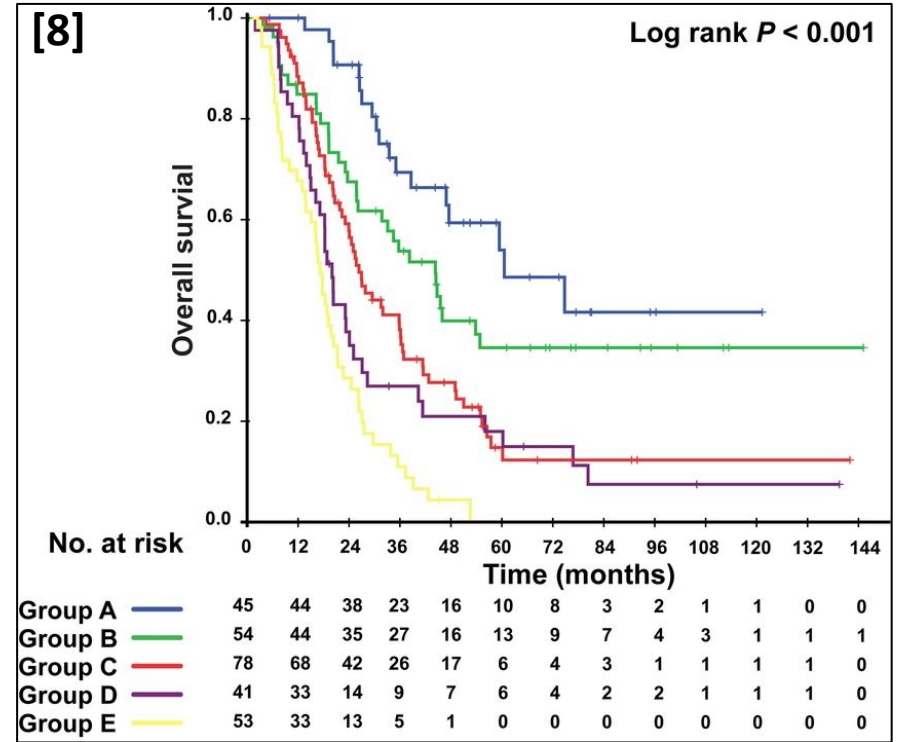
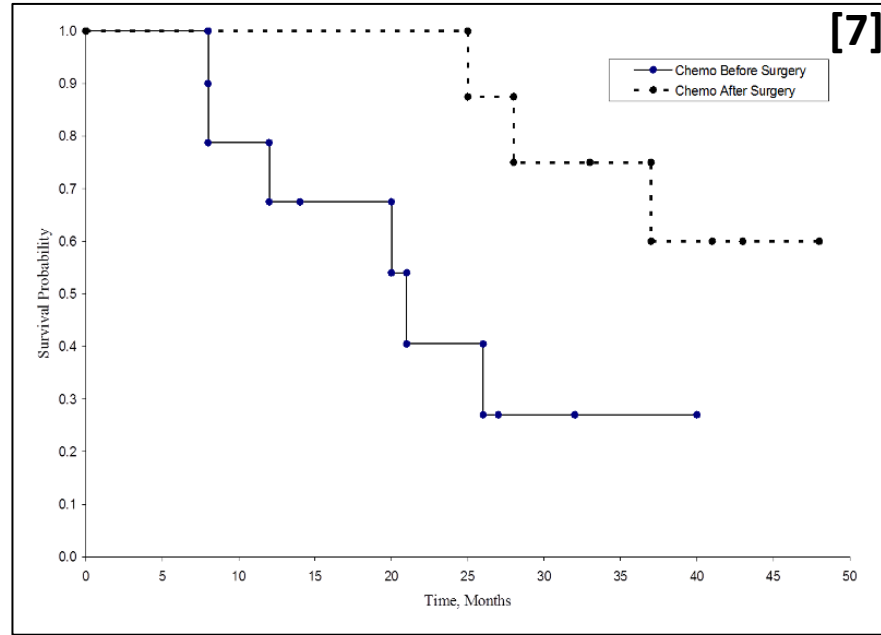


Structure and Uses 4

Survival times compared across groups

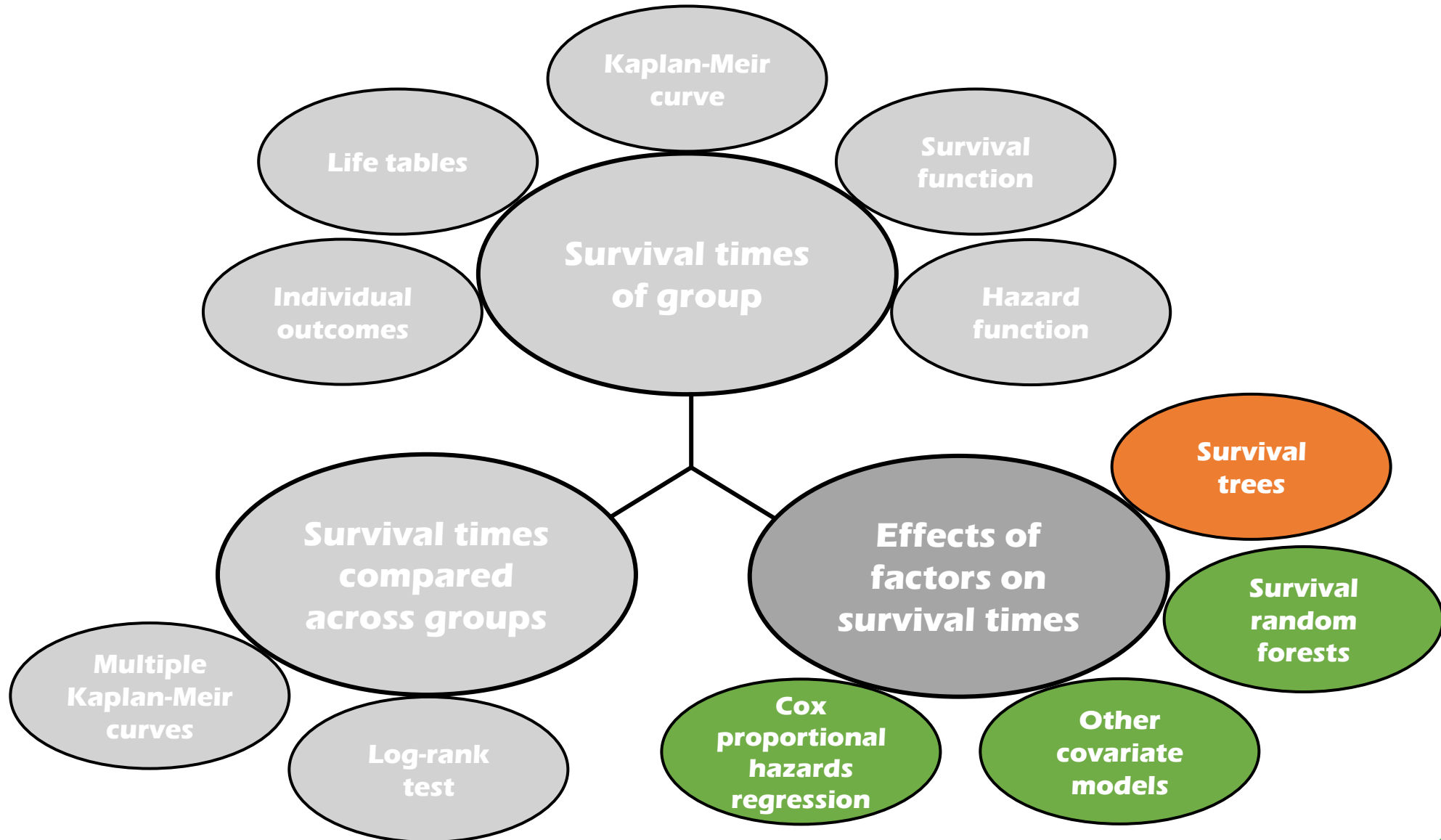
Multiple Kaplan-Meier curves

Log-rank test



$$\text{Log-rank statistic for two groups} = \frac{(O_2 - E_2)^2}{\text{Var}(O_2 - E_2)} \sim \chi_1^2 \quad [9]$$

Structure and Uses 5



Structure and Uses 6

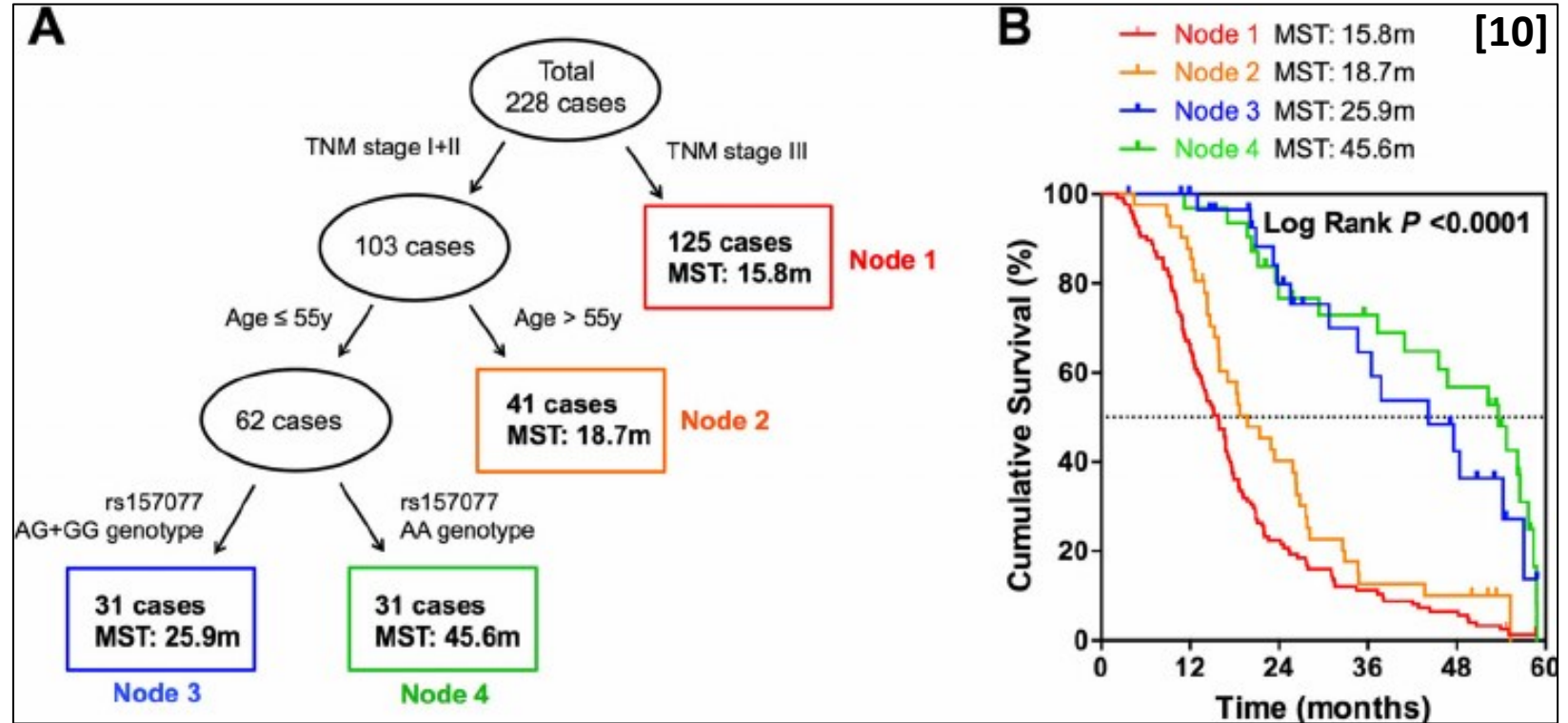
Effects of factors on survival times

Survival trees

Survival random forests

Cox proportional hazards regression

Other covariate models



Structure and Uses 6

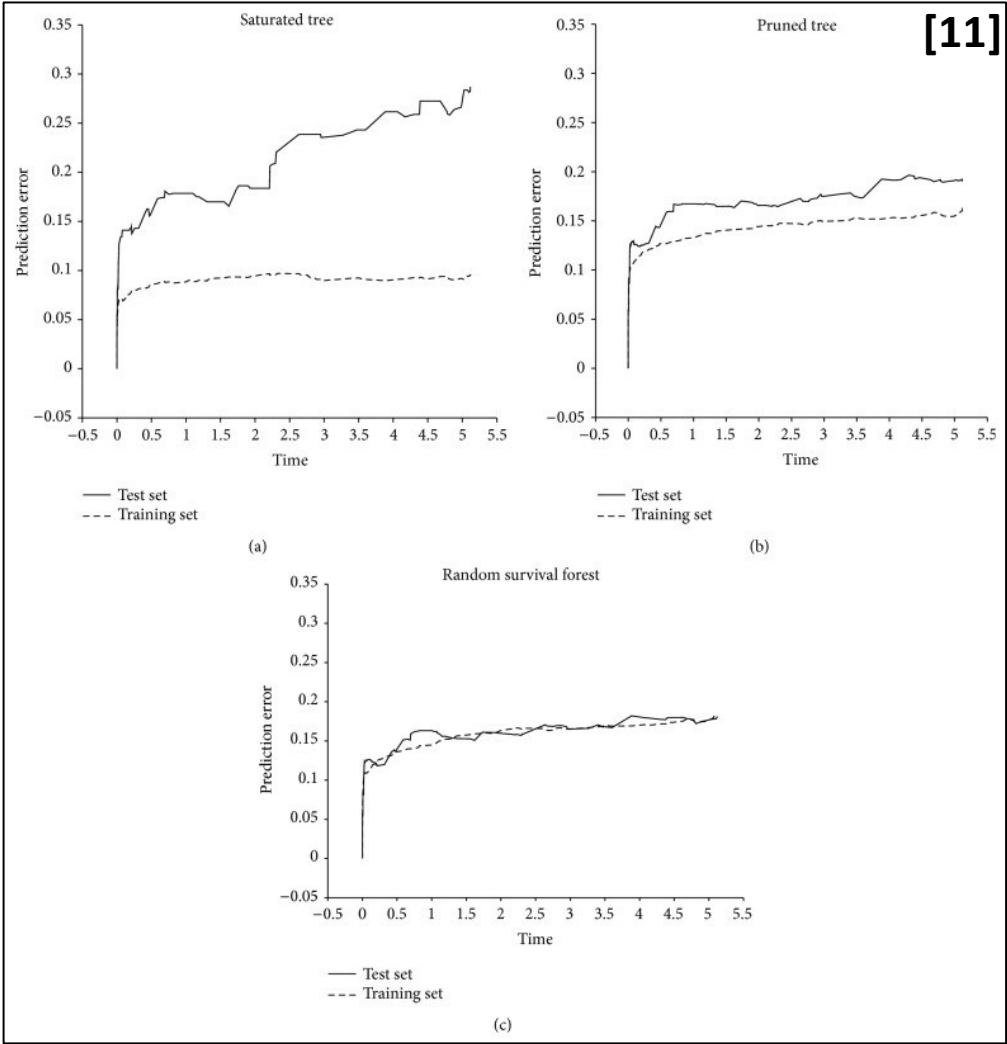
Effects of factors on survival times

Survival trees

Survival random forests

Cox proportional hazards regression

Other covariate models



Structure and Uses 6

Effects of factors on survival times

Survival trees

Survival random forests

Cox proportional hazards regression

Other covariate models

Risk Factor	Parameter Estimate	P-Value	Hazard Ratio (HR) (95% CI for HR)
Age, years	0.11691	0.0001	1.124 (1.111-1.138)
Male Sex	0.40359	0.0002	1.497 (1.215-1.845)
Systolic Blood Pressure	0.01645	0.0001	1.017 (1.012-1.021)
Current Smoker	0.76798	0.0001	2.155 (1.758-2.643)
Total Serum Cholesterol	-0.00209	0.0963	0.998 (0.995-2.643)
Diabetes	-0.02366	0.1585	0.816 (0.615-1.083)

[12]

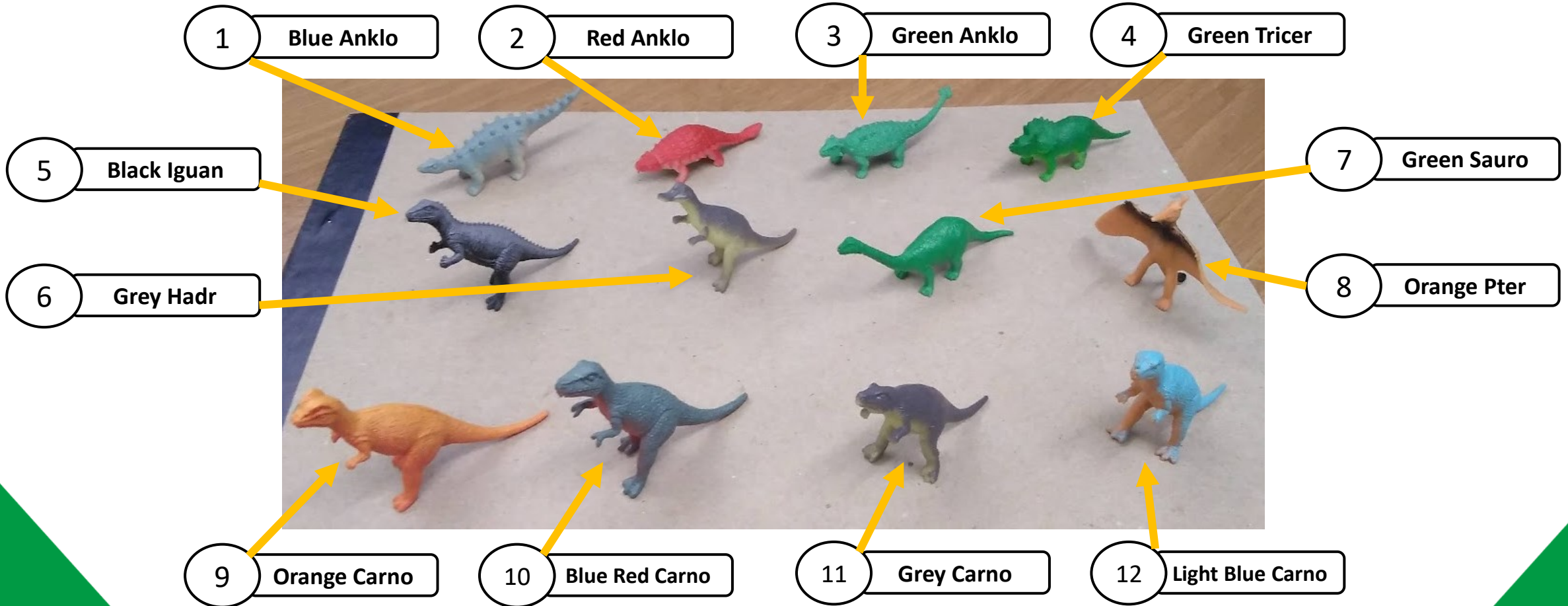
There are four ways to introduce covariates in parametric survival models

- ➊ Parametric families, where the parameters of a distribution, such as λ and p in a Weibull, depend on covariates
- ➋ Accelerated life, where the log of survival time follows a linear model
- ➌ Proportional hazards, where the log of the hazard function follows a linear model
- ➍ Proportional odds, where the logit of the survival function follows a linear model

[13]

Examples

Generating Data



Examples

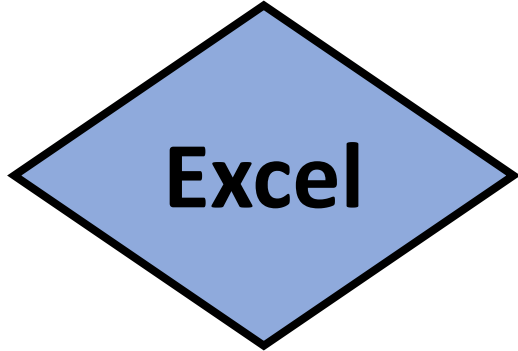
Generating Data 2

Shaking for 60 seconds. Falling off means counted as Dead (1).



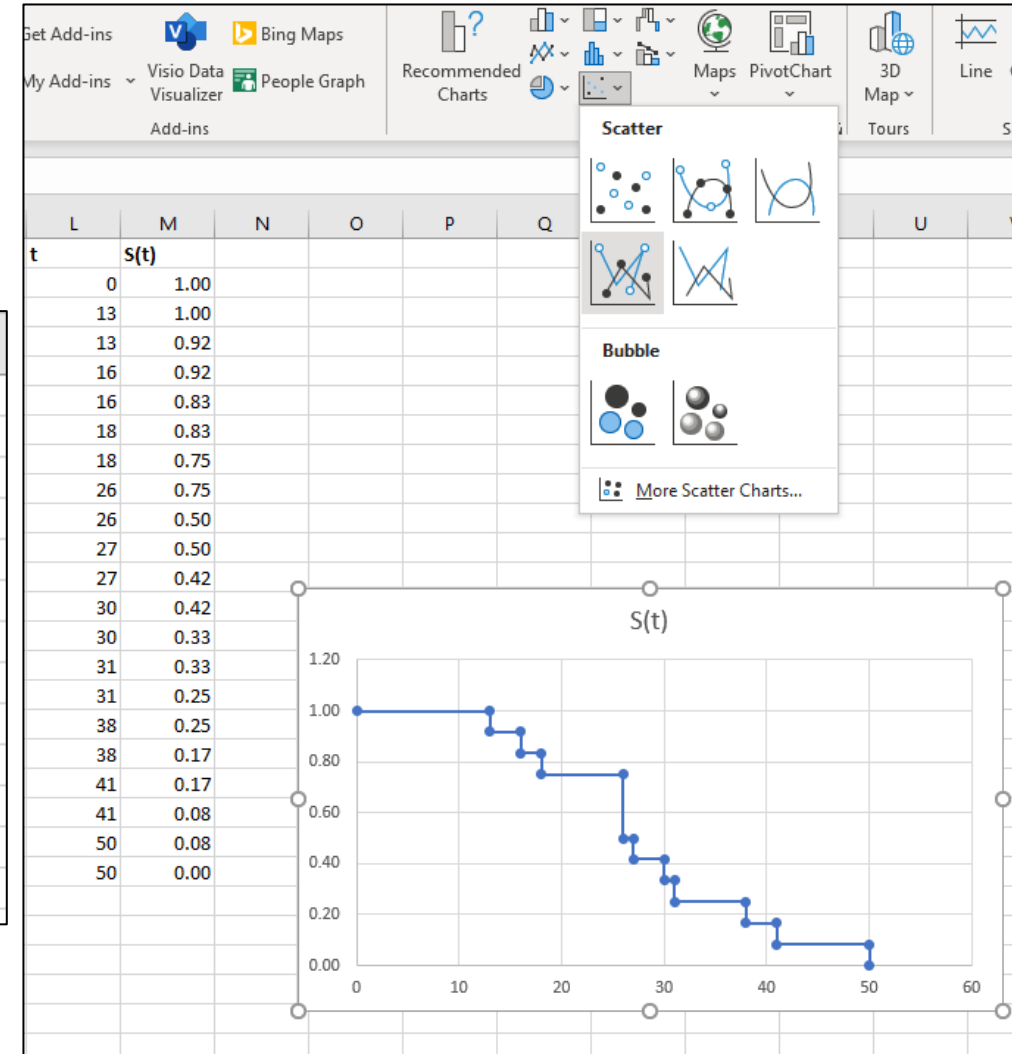
Time	Dead	N
0		12
13	1	12
16	1	11
18	1	10
26	3	9
27	1	6
30	1	5
31	1	4
38	1	3
41	1	2
50	1	1
60	0	0

Examples



	A	B	C	D
1	type	id	time	status
2	Blue Anklo	1	26	1
3	Red Anklo	2	26	1
4	Green Anklo	3	31	1
5	Green Tricer	4	18	1
6	Black Iguan	5	38	1
7	Grey Hadr	6	13	1
8	Green Sauro	7	30	1
9	Orang Pter	8	26	1
10	Orange Carno	9	50	1
11	Blue Red Carno	10	27	1
12	Grey Carno	11	16	1
13	Light Blue Carno	12	41	1

F	G	H	I	J
Time	Dead	N	1-d/n	S(t)
0		12		1
13	1	12	0.916667	0.916667
16	1	11	0.909091	0.833333
18	1	10	0.9	0.75
26	3	9	0.666667	0.5
27	1	6	0.833333	0.416667
30	1	5	0.8	0.333333
31	1	4	0.75	0.25
38	1	3	0.666667	0.166667
41	1	2	0.5	0.083333
50	1	1	0	0



Examples

R

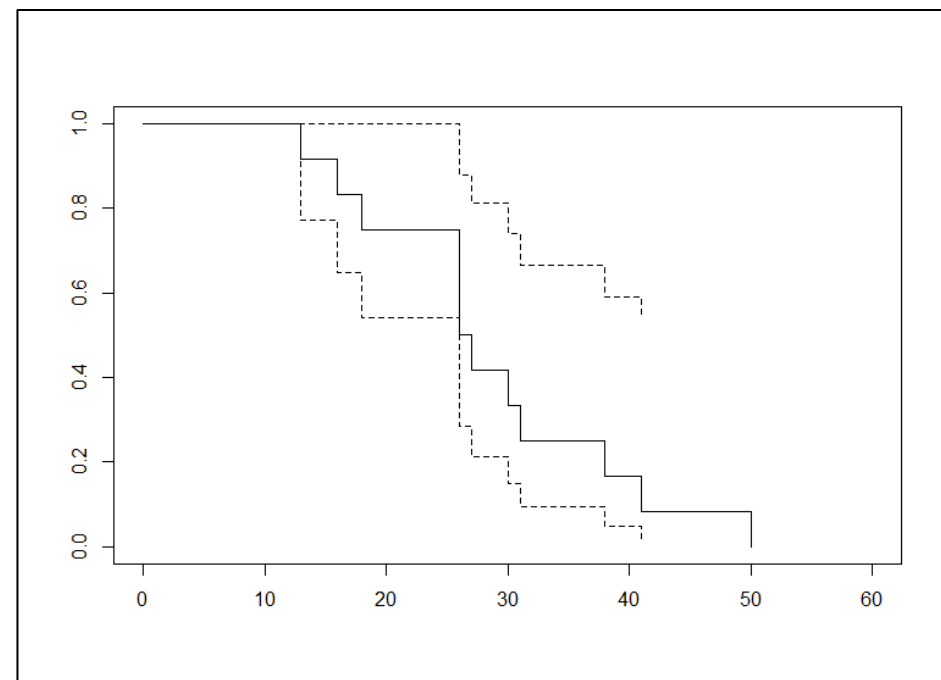
```
#Load and View Data
library("survival")
setwd("C:/Users/Mark.williamson.2/Desktop/williamson Data/R/R_data")
Dino <-read.csv("Dino_Survival.csv")
print(Dino)
```

```
#Survival Curve
dinofit <-survfit(Surv(time, status==1)~1,
                 data=Dino)
summary(dinofit)
plot(dinofit, xlim=c(0,60))
```

```
#Smooth Survival Curve Attempt 1
dinofit2 <-survreg(Surv(time, status==1)~1,
                  data=Dino, dist='exponential')
summary(dinofit2) #3.350
T<-seq(0,60,0.1)
plot(dinofit, xlim=c(0,60))
points(T, 1-pexp(T, exp(-3.350)),
       cex=0.3, pch=16, col="red")
```

```
#Smooth Survival Curve Attempt 3
dinofit3 <-survreg(Surv(time, status==1)~1,
                  data=Dino, dist='weibull', scale=0)
summary(dinofit3) #3.464; 0.331
plot(dinofit, xlim=c(0,60))
points(T, 1-pweibull(T,1/0.331, exp(3.464)),
       cex=0.3, pch=16, col="purple")
```

	type	id	time	status
1	Blue_Anklo	1	26	1
2	Red_Anklo	2	26	1
3	Green_Anklo	3	31	1
4	Green_Tricer	4	18	1
5	Black_Iguan	5	38	1
6	Grey_Hadr	6	13	1
7	Green_Sauro	7	30	1
8	Orang_Pter	8	26	1
9	Orange_Carno	9	50	1
10	Blue_Red_Carno	10	27	1
11	Grey_Carno	11	16	1
12	Light_Blue_Carno	12	41	1



Examples

R

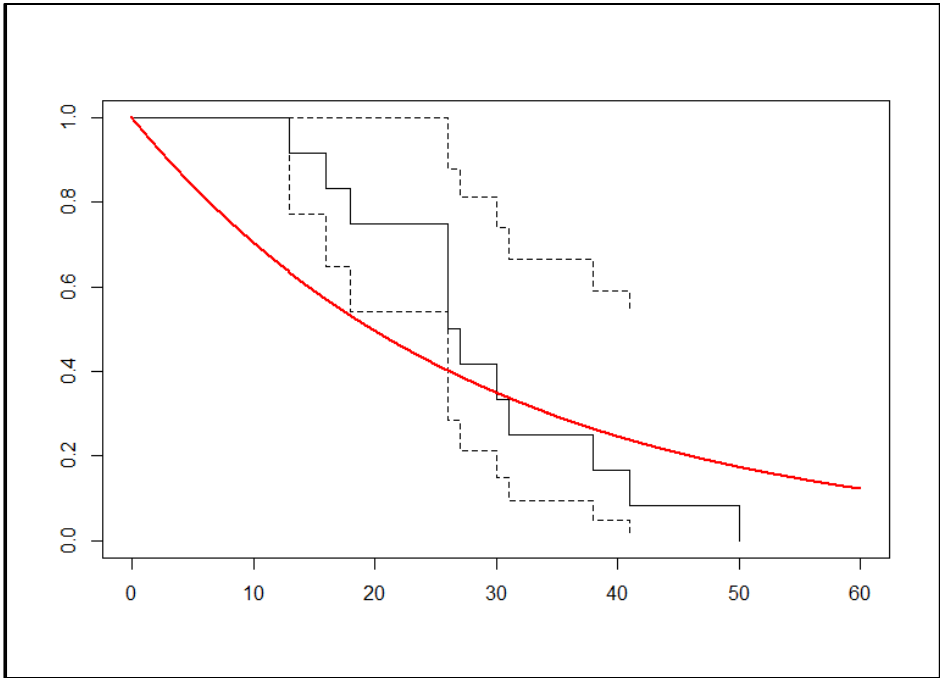
```
#Load and View Data
library("survival")
setwd("C:/Users/Mark.williamson.2/Desktop/williamson Data/R/R_data")
Dino <-read.csv("Dino_Survival.csv")
print(Dino)
```

```
#Survival Curve
dinofit <-survfit(Surv(time, status==1)~1,
                 data=Dino)
summary(dinofit)
plot(dinofit, xlim=c(0,60))
```

```
#Smooth Survival Curve Attempt 1
dinofit2 <-survreg(Surv(time, status==1)~1,
                  data=Dino, dist='exponential')
summary(dinofit2) #3.350
T<-seq(0,60,0.1)
plot(dinofit, xlim=c(0,60))
points(T, 1-pexp(T, exp(-3.350)),
       cex=0.3, pch=16, col="red")
```

```
#Smooth Survival Curve Attempt 3
dinofit3 <-survreg(Surv(time, status==1)~1,
                  data=Dino, dist='weibull', scale=0)
summary(dinofit3) #3.464; 0.331
plot(dinofit, xlim=c(0,60))
points(T, 1-pweibull(T,1/0.331, exp(3.464)),
       cex=0.3, pch=16, col="purple")
```

	type	id	time	status
1	Blue_Anklo	1	26	1
2	Red_Anklo	2	26	1
3	Green_Anklo	3	31	1
4	Green_Tricer	4	18	1
5	Black_Iguan	5	38	1
6	Grey_Hadr	6	13	1
7	Green_Sauro	7	30	1
8	Orang_Pter	8	26	1
9	Orange_Carno	9	50	1
10	Blue_Red_Carno	10	27	1
11	Grey_Carno	11	16	1
12	Light_Blue_Carno	12	41	1



Examples

R

```
#Load and View Data
library("survival")
setwd("C:/Users/Mark.williamson.2/Desktop/williamson Data/R/R_data")
Dino <-read.csv("Dino_Survival.csv")
print(Dino)
```

```
#Survival Curve
dinofit <-survfit(Surv(time, status==1)~1,
                 data=Dino)
summary(dinofit)
plot(dinofit, xlim=c(0,60))
```

```
#Smooth Survival Curve Attempt 1
dinofit2 <-survreg(Surv(time, status==1)~1,
                  data=Dino, dist='exponential')
summary(dinofit2) #3.350

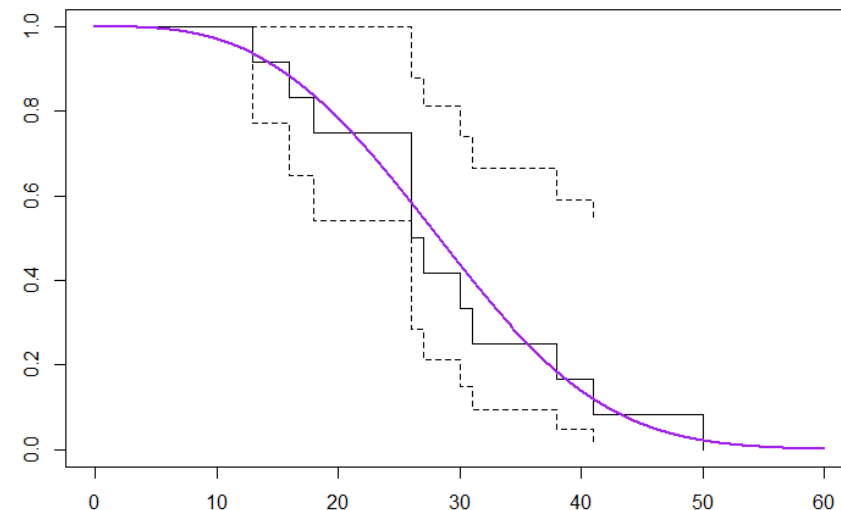
T<-seq(0,60,0.1)

plot(dinofit, xlim=c(0,60))
points(T, 1-pexp(T, exp(-3.350)),
       cex=0.3, pch=16, col="red")
```

```
#Smooth Survival Curve Attempt 3
dinofit3 <-survreg(Surv(time, status==1)~1,
                  data=Dino, dist='weibull', scale=0)
summary(dinofit3) #3.464; 0.331

plot(dinofit, xlim=c(0,60))
points(T, 1-pweibull(T,1/0.331, exp(3.464)),
       cex=0.3, pch=16, col="purple")
```

	type	id	time	status
1	Blue_Anklo	1	26	1
2	Red_Anklo	2	26	1
3	Green_Anklo	3	31	1
4	Green_Tricer	4	18	1
5	Black_Iguan	5	38	1
6	Grey_Hadr	6	13	1
7	Green_Sauro	7	30	1
8	Orang_Pter	8	26	1
9	Orange_Carno	9	50	1
10	Blue_Red_Carno	10	27	1
11	Grey_Carno	11	16	1
12	Light_Blue_Carno	12	41	1



Examples



```
*Load and Format Data;
PROC IMPORT datafile='/home/markwilliamson20/my_courses/markwilliamson0/MW_Datasets_2021/Presentation and Module Datasets 2021/Dino_Survival.csv'
  dbms=csv out=dino replace; getnames=yes; guessingrows=10000;
DATA dino; set dino;
  if id >7 then do; diet="Carnivore"; end;
  else do; diet="Herbivore"; end;
PROC PRINT data=dino;
```

```
*Hazard Function;
PROC LIFETEST data=dino plots=hazard;
  time time*status(0);
```

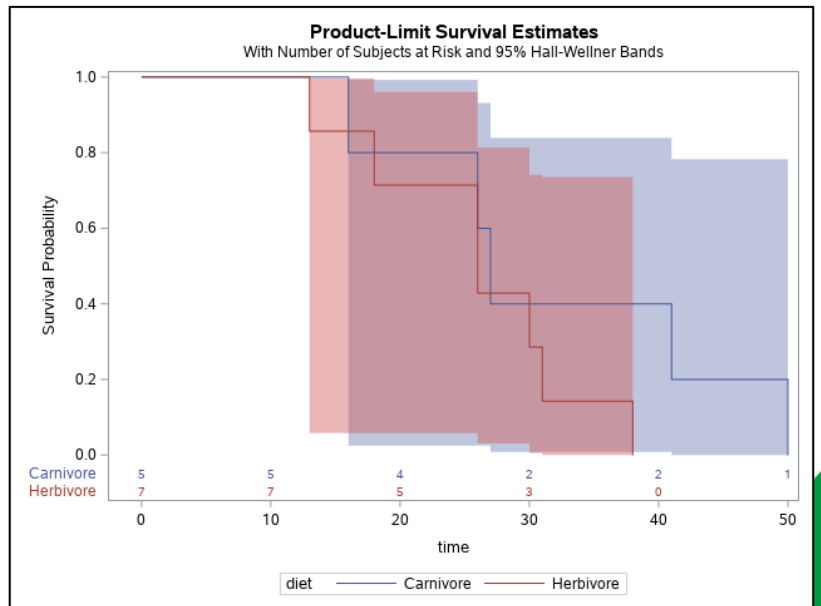
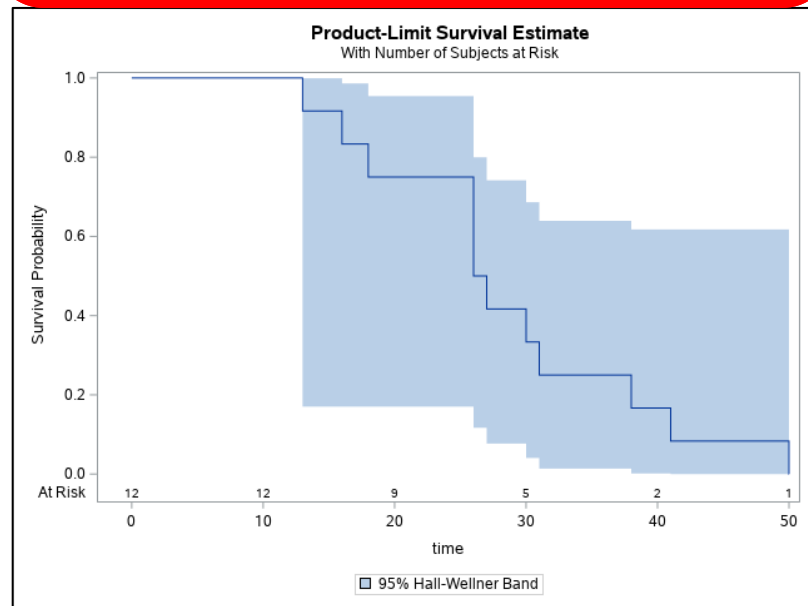
```
*Cumulative Hazard Function;
PROC LIFETEST data=dino nelson method=pl;
  time time*status(0);
  ods output ProductLimitEstimates=dino_ple;
PROC PRINT data=dino_ple;
PROC SGPLOT data =dino_ple;
  series x=time y=CumHaz;
```

```
*Survival Function;
PROC LIFETEST data=dino plots=survival(atrisk cb);
  time time*status(0);

*Survival Function (two groups);
PROC LIFETEST data=dino plots=survival(atrisk cb);
  strata diet;
  time time*status(0);
```

Obs	type	id	time	status	diet
1	Orang_Pter	8	26	1	Carnivore
2	Orange_Carno	9	50	1	Carnivore
3	Blue_Red_Carno	10	27	1	Carnivore
4	Grey_Carno	11	16	1	Carnivore
5	Light_Blue_Carno	12	41	1	Carnivore
6	Blue_Anklo	1	26	1	Herbivore
7	Red_Anklo	2	26	1	Herbivore
8	Green_Anklo	3	31	1	Herbivore
9	Green_Tricer	4	18	1	Herbivore
10	Black_Iguan	5	38	1	Herbivore
11	Grey_Hadr	6	13	1	Herbivore
12	Green_Sauro	7	30	1	Herbivore

Test of Equality over Strata			
Test	Chi-Square	DF	Pr > Chi-Square
Log-Rank	1.2420	1	0.2651
Wilcoxon	0.5226	1	0.4697
-2Log(LR)	0.1270	1	0.7216



Examples



```
*Load and Format Data;
PROC IMPORT datafile='/home/markwilliamson20/my_courses/markwilliamson0/MW_Datasets_2021/Presentation and Module Datasets 2021/Dino_Survival.csv'
  dbms=csv out=dino replace; getnames=yes; guessingrows=10000;
DATA dino; set dino;
  if id >7 then do; diet="Carnivore"; end;
  else do; diet="Herbivore"; end;
PROC PRINT data=dino;
```

```
*Survival Function;
PROC LIFETEST data=dino plots=survival(atrisk cb);
time time*status(0);
```

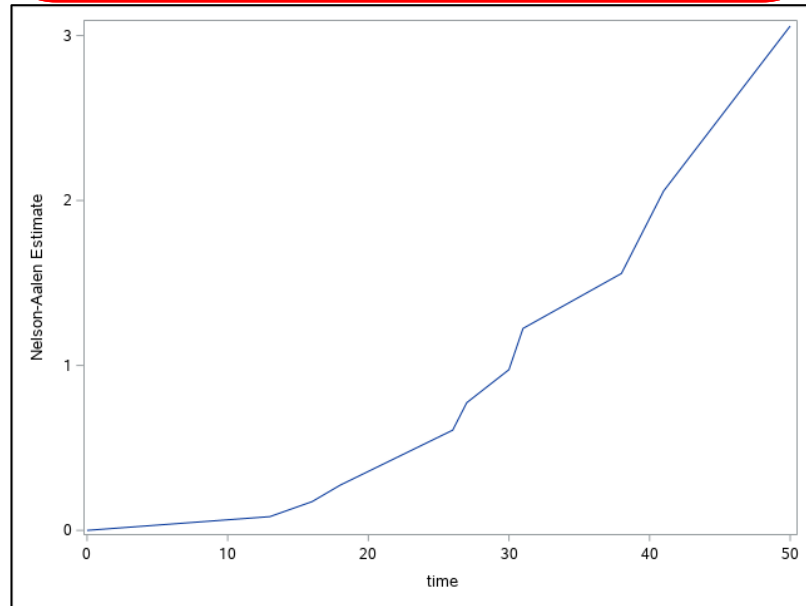
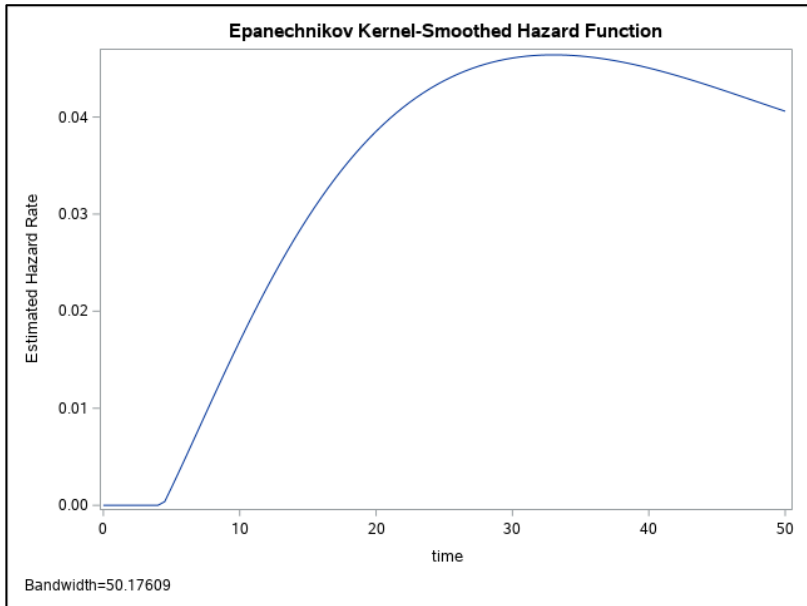
```
*Survival Function (two groups);
PROC LIFETEST data=dino plots=survival(atrisk cb);
strata diet;
time time*status(0);
```

```
*Hazard Function;
PROC LIFETEST data=dino plots=hazard;
time time*status(0);
```

```
*Culmulative Hazard Function;
PROC LIFETEST data=dino nelson method=pl;
time time*status(0);
ods output ProductLimitEstimates=dino_ple;
PROC PRINT data=dino_ple;
PROC SGPLOT data =dino_ple;
series x=time y=CumHaz;
```

Obs	type	id	time	status	diet
1	Orang_Pter	8	26	1	Carnivore
2	Orange_Carno	9	50	1	Carnivore
3	Blue_Red_Carno	10	27	1	Carnivore
4	Grey_Carno	11	16	1	Carnivore
5	Light_Blue_Carno	12	41	1	Carnivore
6	Blue_Anklo	1	26	1	Herbivore
7	Red_Anklo	2	26	1	Herbivore
8	Green_Anklo	3	31	1	Herbivore
9	Green_Tricer	4	18	1	Herbivore
10	Black_Iguan	5	38	1	Herbivore
11	Grey_Hadr	6	13	1	Herbivore
12	Green_Sauro	7	30	1	Herbivore

Test of Equality over Strata			
Test	Chi-Square	DF	Pr > Chi-Square
Log-Rank	1.2420	1	0.2651
Wilcoxon	0.5226	1	0.4697
-2Log(LR)	0.1270	1	0.7216



Assessment

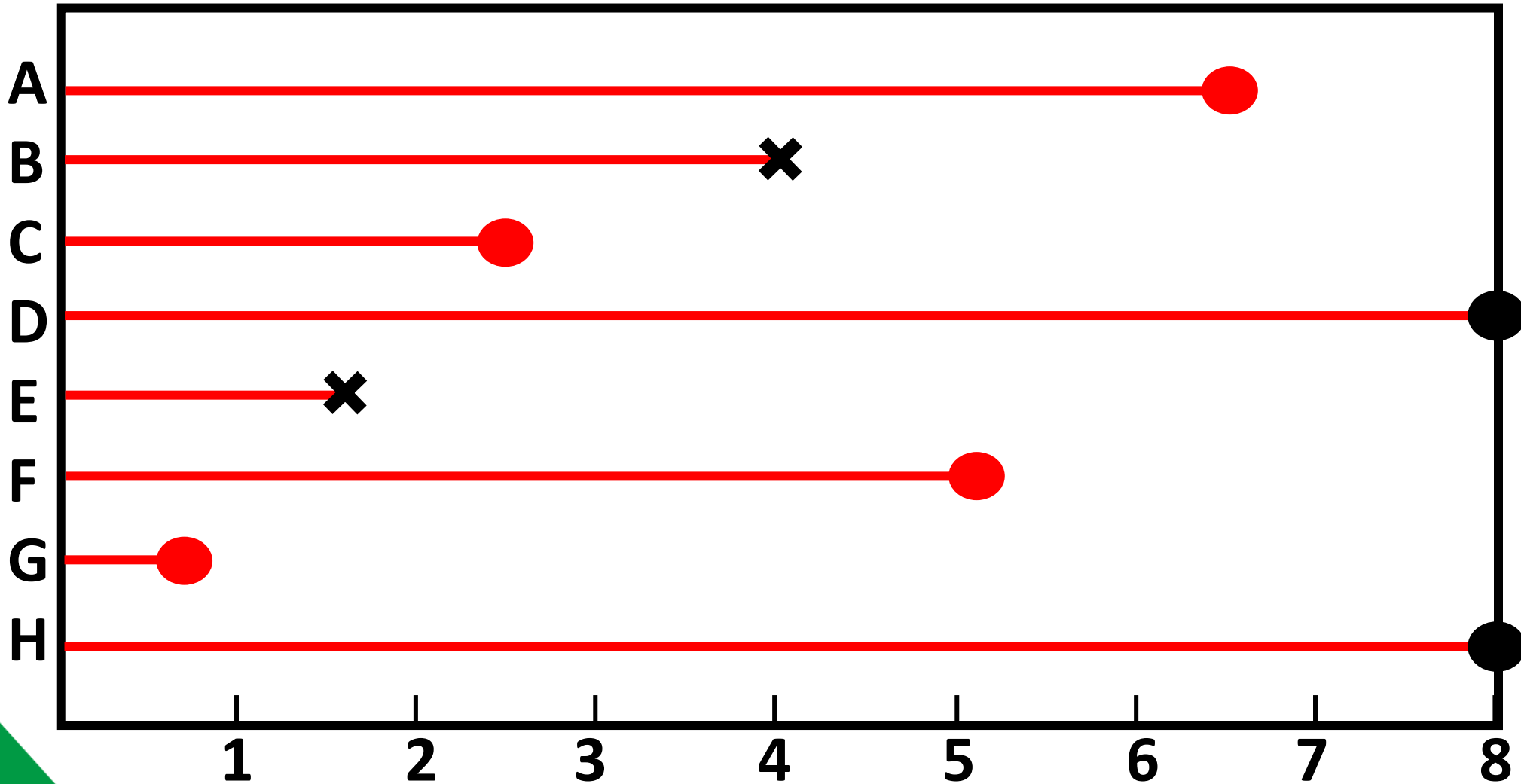


qualtrics^{XM}

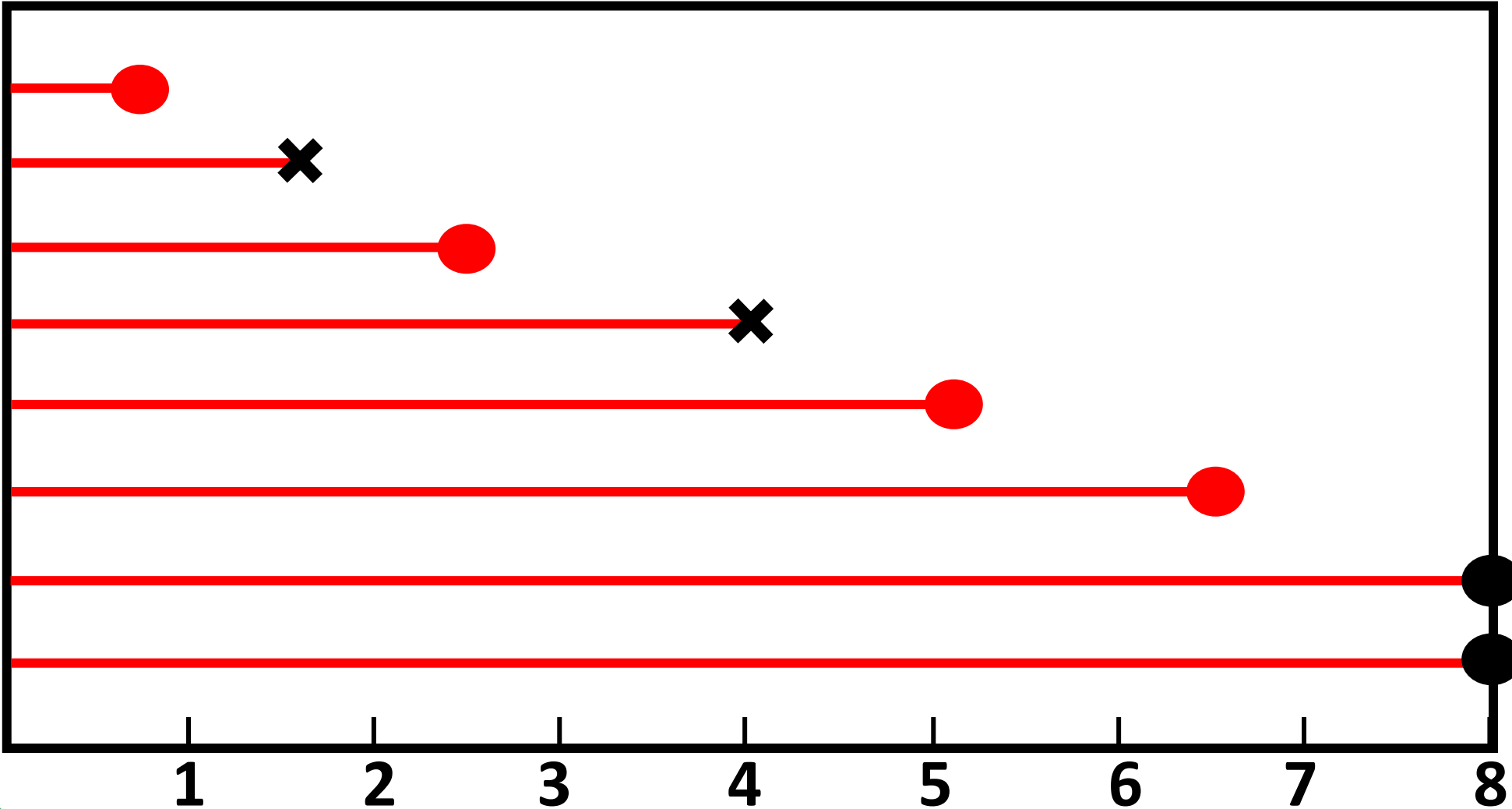


https://und.qualtrics.com/jfe/form/SV_3Q4QXbdVLMGTva6

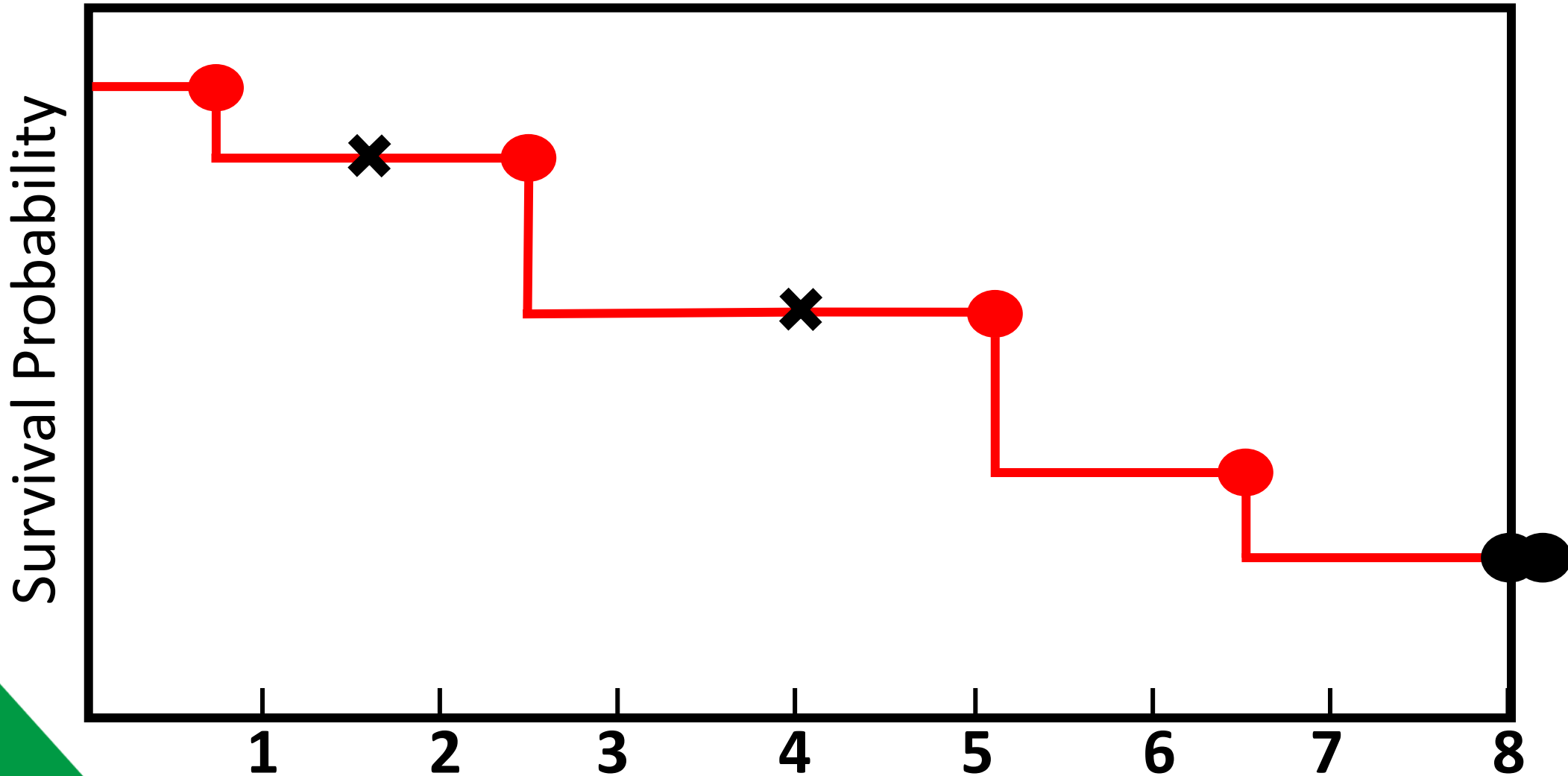
An Illustration



An Illustration



An Illustration



Summary and Conclusion



- Survival analysis is used for time-to-event data
- Major types are 1) survival time of group, 2) survival time across groups, and 3) survival time based on factors
- Survival curves and life tables are common visualization methods
- Log-rank tests and Cox regressions are common analysis methods
- Tune in next time for a more detailed look in Survival Analysis Module II: Leaves and Trees

References

- [1] https://en.wikipedia.org/wiki/Survival_analysis
- [2] <http://www.stat.columbia.edu/~madigan/W2025/notes/survival.pdf>
- [3] <https://www.real-statistics.com/survival-analysis/survival-analysis-basic-concepts/>
- [4] https://www.emilyzabor.com/tutorials/survival_analysis_in_r_tutorial.html
- [5] <https://www.real-statistics.com/survival-analysis/kaplan-meier-procedure/kaplan-meier-overview/>
- [6] <https://www.real-statistics.com/survival-analysis/kaplan-meier-procedure/survival-curve/>
- [7] https://sphweb.bumc.bu.edu/otlt/MPH-Modules/BS/BS704_survival/BS704_Survival5.html
- [8] https://www.researchgate.net/publication/340989649_Establishment_of_an_integrated_model_for_predicting_survival_and_guiding_treatment_in_local_recurrence_nasopharyngeal_carcinoma/figures?lo=1
- [9] <https://www.slideshare.net/zhe1/kaplan-meier-survival-curves-and-the-logrank-test>
- [10] https://www.researchgate.net/publication/274396208_Glutathione_S-transferase_O2_gene_rs157077_polymorphism_predicts_response_to_transarterial_chemoembolization_in_hepatocellular_carcinoma/figures?lo=1
- [11] <https://pubmed.ncbi.nlm.nih.gov/26858773/#&gid=article-figures&pid=figure-1-uid-0>
- [12] https://sphweb.bumc.bu.edu/otlt/MPH-Modules/BS/BS704_Survival/BS704_Survival6.html
- [13] <https://data.princeton.edu/pop509/pop509slides1.pdf>

Acknowledgements

- The DaCCoTA is supported by the National Institute of General Medical Sciences of the National Institutes of Health under Award Number U54GM128729.
- For the labs that use the Biostatistics, Epidemiology, and Research Design Core in any way, including this Module, please acknowledge us for publications. ***"Research reported in this publication was supported by DaCCoTA (the National Institute of General Medical Sciences of the National Institutes of Health under Award Number U54GM128729)".***

DaCCoTA
DAKOTA CANCER COLLABORATIVE
ON TRANSLATIONAL ACTIVITY