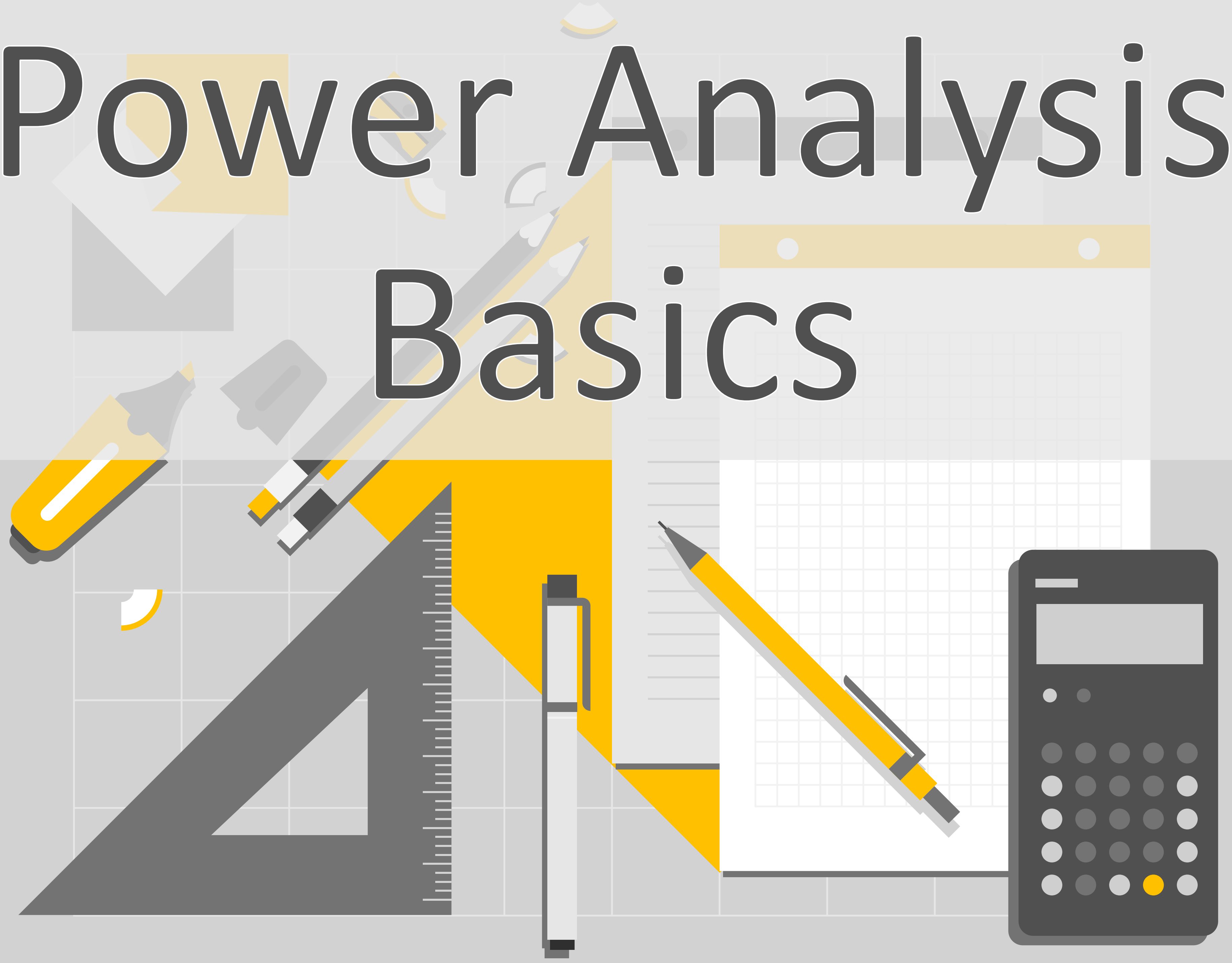
Power Analysis Basics



Dr. Mark Williamson

BERDC UND

1. What is Power Analysis?

- Power is the probability of detecting an effect, given that the effect is truly there [1]
  - Example, the effect of a treatment, measured by comparing the mean between a treatment group and a control group
- The most common use of power analysis is to determine sample sizes for experiments
  - Too few, you could miss a true effect
  - Too many, you could waste resources or unethically expose more patients to risk
- Requirements for some studies

2. Components needed.

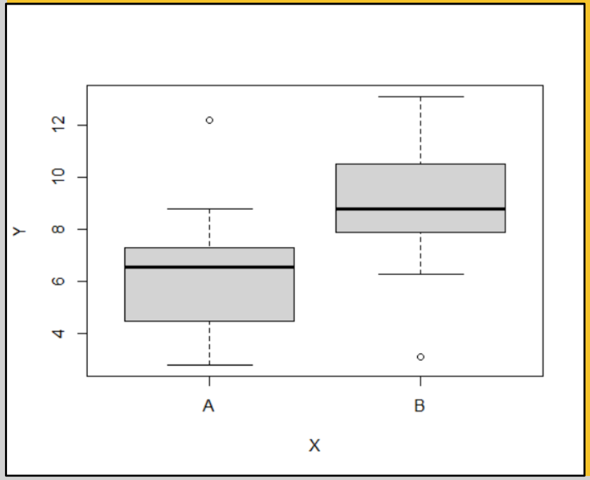
- In a basic power analysis, there are four components [2]:
  - Alpha (α) = risk of a Type I error (false positive)
  - Power = 1 – risk of a Type II error (false negative)
  - Sample size (n) = number of samples needed to detect the effect
  - Effect size = magnitude of the effect under the alternative hypothesis
- Alpha and Power have defaults:
  - Alpha = 0.05
  - Power = 0.80
- Effect size is the major unknown:
  - Can be estimated
  - Can be guessed

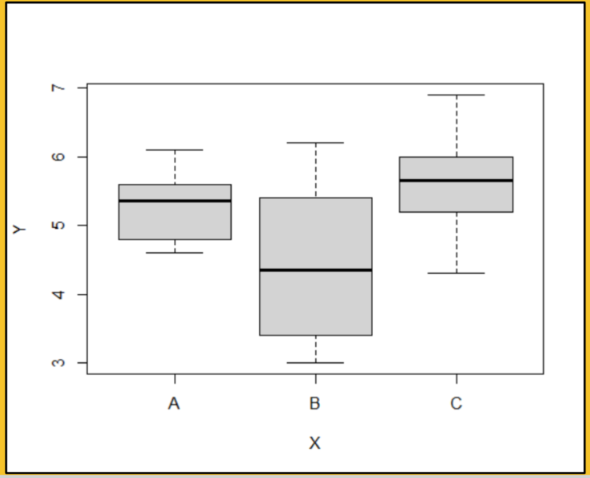
3. The Challenge of Effect Size.

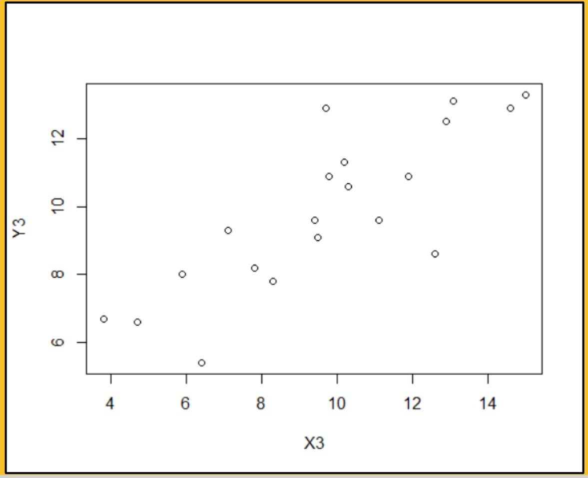
- Effect size matters the most for sample size
  - The higher the effect size, the fewer samples needed
- Calculating Effect size:
  - Prior studies (pilot, trial, etc.)
  - Similar studies (same or related field)
  - Other pertinent background information
- Guessing Effect size:
  - Rules of thumb for small, medium, large effects
  - Enter plausible values to effect size equation

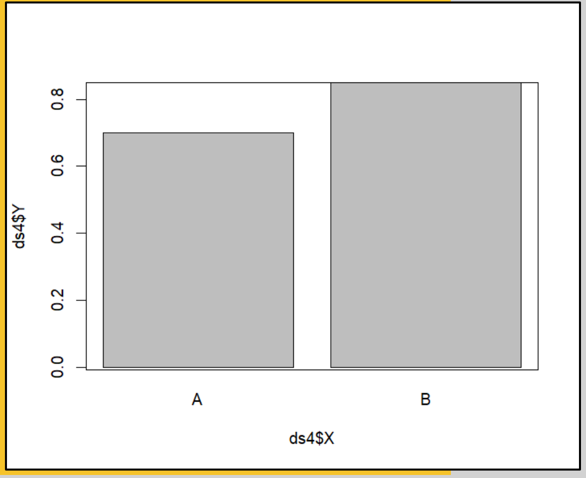
4. Basic Comparisons.

- Comparing 2 groups (T-test)
- Comparing 3+ groups (ANOVA)
- Correlation
- Proportions









5. Guessing Effect Size.

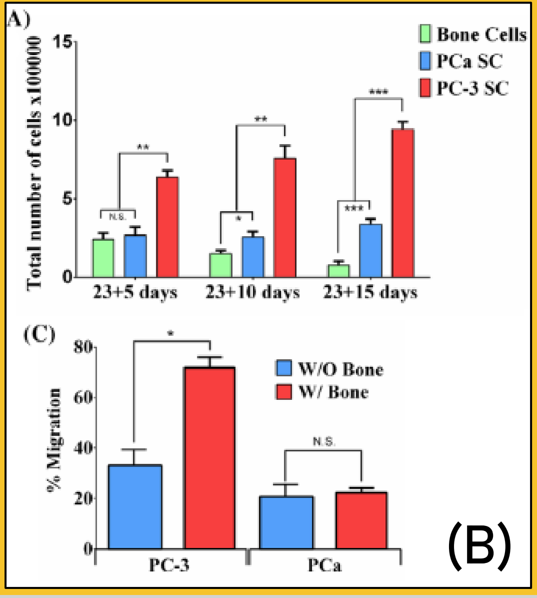
- Rule of thumb values [3,4]
- Enter plausible equation values
  - T-test: Two-fold difference, SD 1/2 of mean

$$d = \frac{10 - 5}{5} = 1$$

	Effect Size		
Test	**Small**	**Medium**	**Large**
T-test (d)	0.20	0.50	0.80
ANOVA (f)	0.10	0.25	0.40
Correlation (R)	0.10	0.30	0.50

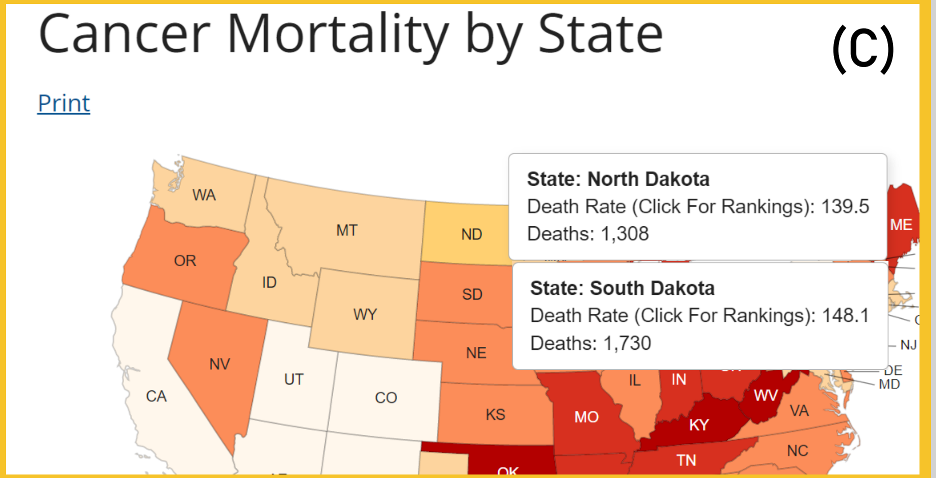
6. Calculating Effect Size.

- Raw data (A)
- Literature search (B)
- Summary statistics (C)



(A)

	Treat	Control
	6.8	4.7
	3.2	3.3
	3.4	5
	5.1	2.5
	7.9	2.4
	7.4	1.9
	8.8	0.6
	4.4	2.6
	5.1	7.2
	8.2	5
mean	6	3.5
SD	2	1.9



7. Tools to Use.

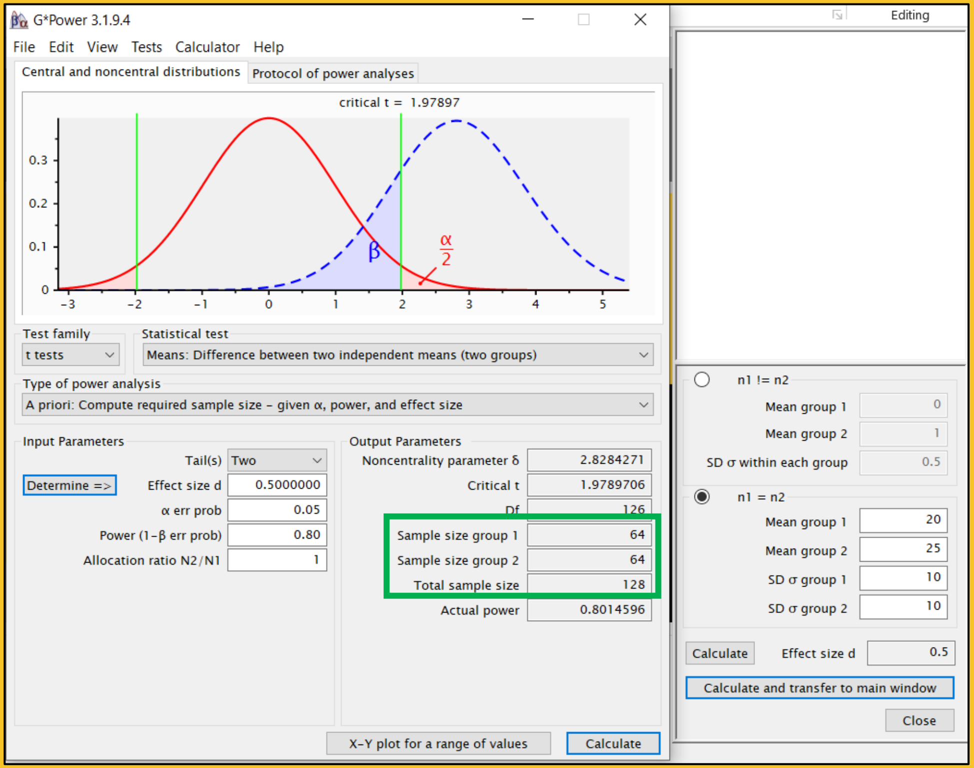
- By hand with equations (!!!)
- G*Power[5]
- Online calculator
  - Powerandsamplesize.com[6]
  - GIGAcalculator[7]
  - Statistics Kingdom[8]
  - Sample-size.net[9]
- Software
  - R: package ‘pwr’
  - SPSS -> Analyze -> Power Analysis

8. T-test Example.

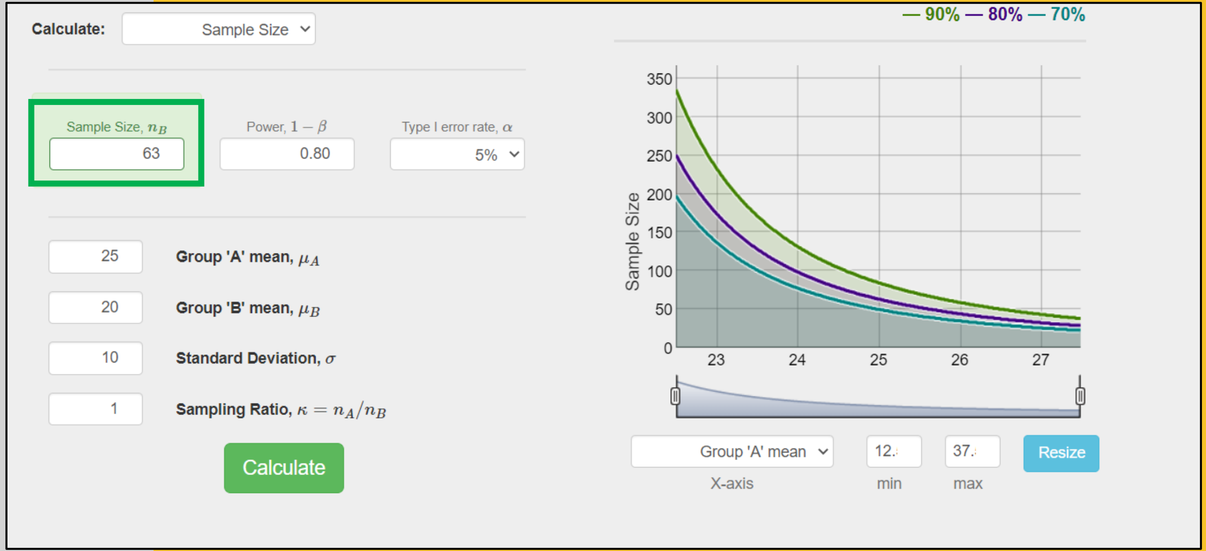
- Data:

	Treat	Control
mean	20	25
std	10	10

- Method:
  - G*Power
  - Online calculator
  - R
  - SPSS

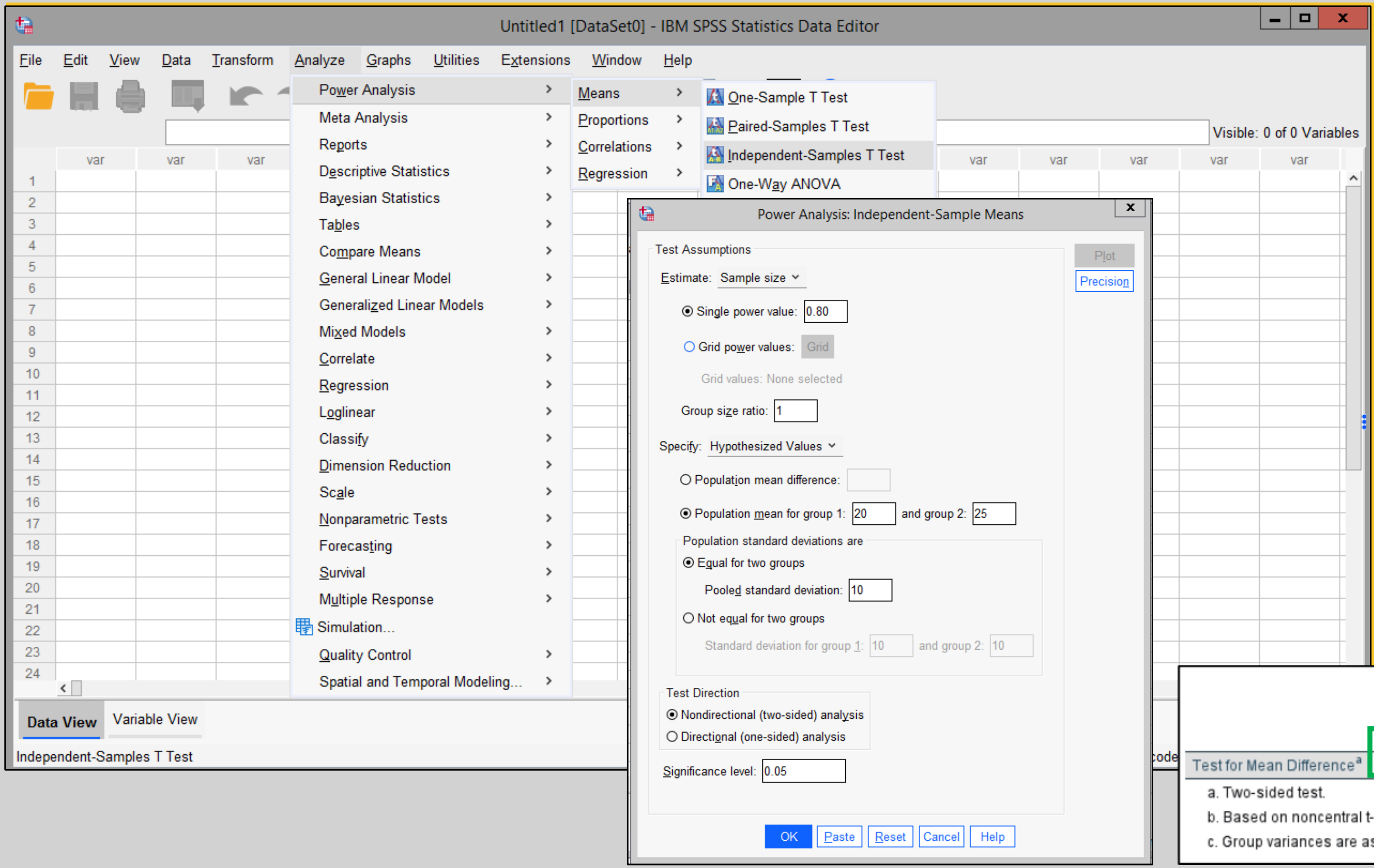


8. T-test Example.




```
R 4.2.1 · ~/
> d <-(25-20)/10
> d
[1] 0.5
> pwr.t.test(d=d, sig.level=0.05 , power=0.8, type=c("two.sample"))

     Two-sample t test power calculation

              n = 63.76561
              d = 0.5
      sig.level = 0.05
          power = 0.8
    alternative = two.sided

NOTE: n is number in *each* group
```

8. T-test Example.



Power Analysis Table

	N1	N2	Actual Power[b]	Test Assumptions: Power	Std. Dev.[c]	Effect Size	Sig.
Test for Mean Difference[a]	64	64	.801	.8	10	.500	.05

a. Two-sided test.

b. Based on noncentral t-distribution.

c. Group variances are assumed to be equal.

9. ANOVA Example.

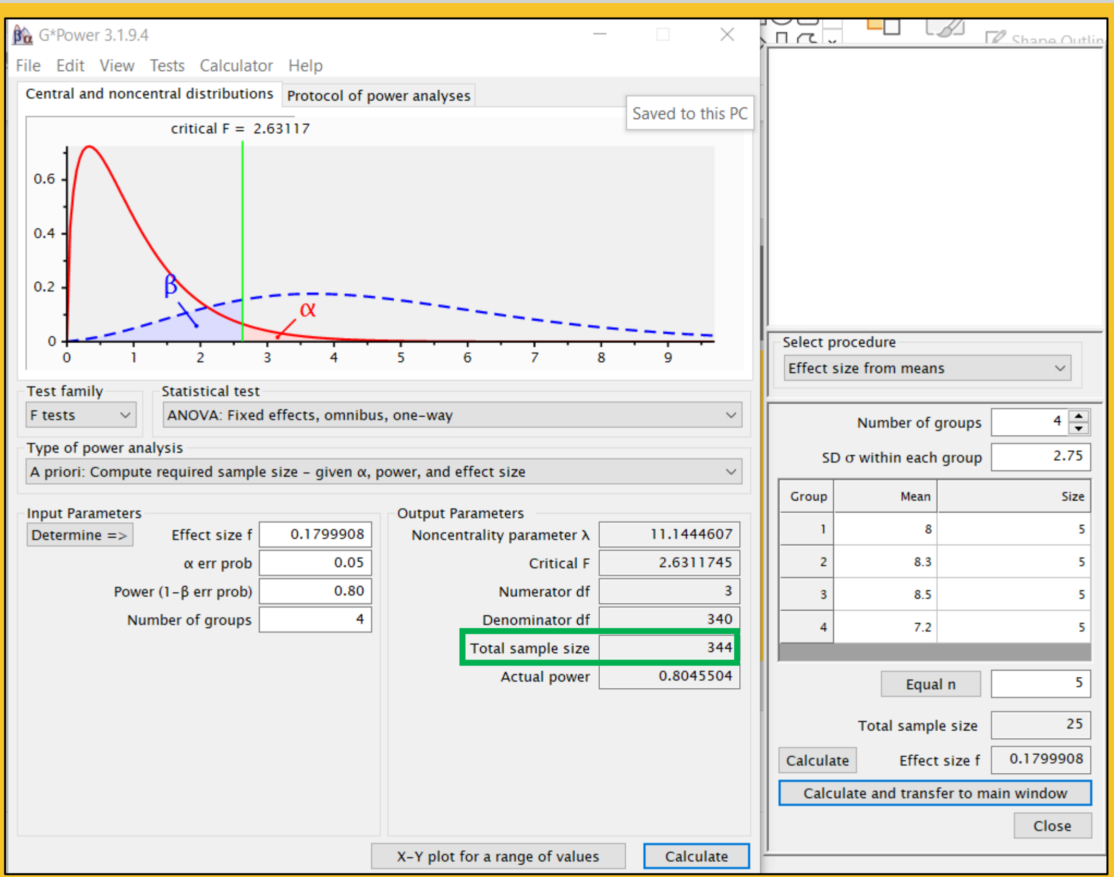
- Data:

	A	B	C	D
mean	8.0	8.3	8.5	7.2
std	2.1	3.2	2.2	3.5

[10, 11]	Df	SS	Mean	F	p
X var	3	6.5	2.163	0.211	0.888
Residuals	36	369.2	2.2		

- Method:
  - G*Power
  - ~~Online calculator~~
  - R
  - SPSS

9. ANOVA Example.



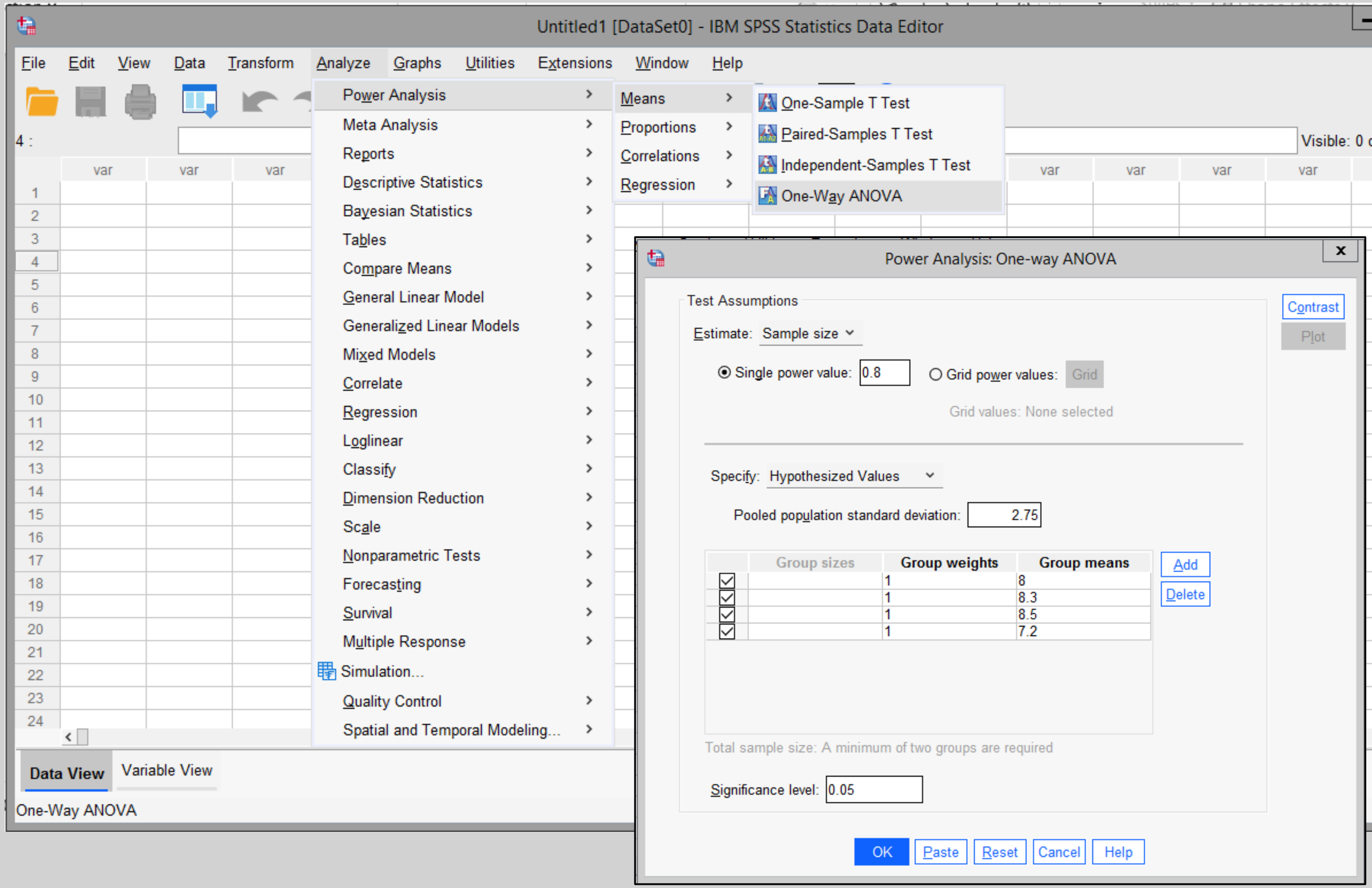
R 4.2.1 · ~/

```
> n2 <- 6.5/(6.5+369.2)
> n2
[1] 0.01730104
> f <-sqrt(n2/(1-n2))
> f
[1] 0.1326862
> pwr.anova.test(k=4 , f=f, sig.level=0.05, power=0.8)

     Balanced one-way analysis of variance power calculation

              k = 4
              n = 155.7967
              f = 0.1326862
      sig.level = 0.05
          power = 0.8

NOTE: n is number in each group

>
> f.2 <-0.179
> pwr.anova.test(k=4 , f=f.2, sig.level=0.05, power=0.8)

     Balanced one-way analysis of variance power calculation

              k = 4
              n = 86.05041
              f = 0.179
      sig.level = 0.05
          power = 0.8

NOTE: n is number in each group
```

Using f calculated from eta squared (η^2) (n = 155.7967)

Using f calculated by G*Power (n = 86.05041)

9. ANOVA Example.



Power Analysis - One-way ANOVA

Power Analysis Table

	N[b]	Actual Power[c]	Test Assumptions Power	Std. Dev.	Effect Size[d]	Sig.
Overall Test[a]	344	.805	.8	2.75	.208	.05

a. Test the null hypothesis that population mean is the same for all groups.
b. Total sample size across groups.
c. Based on noncentral F-distribution.
d. Effect size measured by the root-mean-square standardized effect.

Group Size Allocation for Overall Test

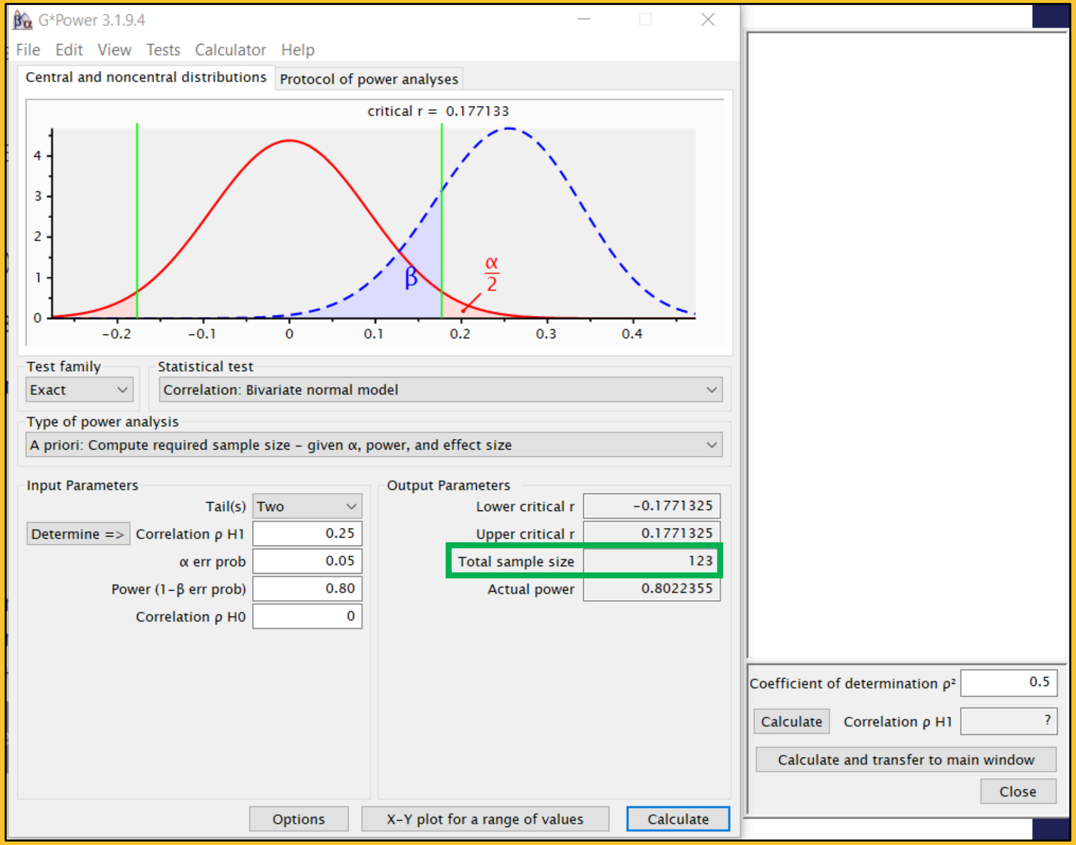
	N
Group 1	86
Group 2	86
Group 3	86
Group 4	86
Overall	344

10. Correlation Example.

- Data:

	H1	H0
Correlation	0.25	0.00

- Method:
  - G*Power
  - Online calculator
  - R
  - SPSS



10. Correlation Example.

Correlation sample size

Total sample size required to determine whether a correlation coefficient differs from zero.

Instructions: Enter parameters in the **green** cells. Answers will appear in the **blue** box below.

α (two-tailed) =	0.05	Threshold probability for rejecting the null hypothesis. Type I error rate.
β =	0.20	Probability of failing to reject the null hypothesis under the alternative hypothesis. Type II error rate.
r =	0.25	The expected correlation coefficient.

Calculate

The standard normal deviate for $\alpha = Z_\alpha = 1.9600$

The standard normal deviate for $\beta = Z_\beta = 0.8416$

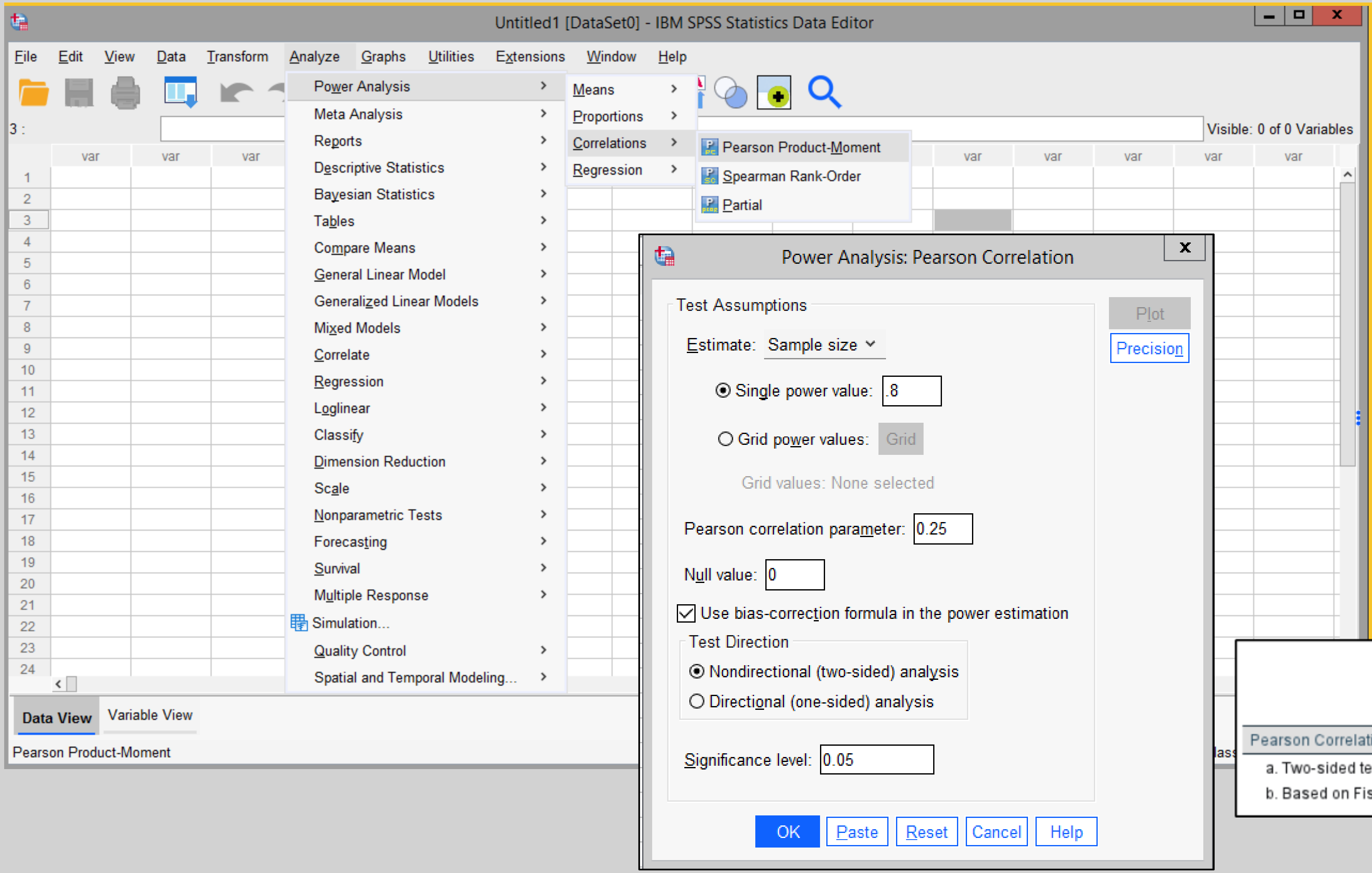
$C = 0.5 * \ln[(1+r)/(1-r)] = 0.2554$

Total sample size = $N = [(Z_\alpha+Z_\beta)/C]^2 + 3 =$ 123

R 4.2.1 · ~/

```
> pwr.r.test(r=0.25, sig.level=0.05, power=0.80)

     approximate correlation power calculation (arctangh transformation)

              n = 122.4466
              r = 0.25
      sig.level = 0.05
          power = 0.8
    alternative = two.sided
```

10. Correlation Example.



Power Analysis Table

	N	Actual Power[b]	Test Assumptions Power	Null	Alternative	Sig.
Pearson Correlation[a]	123	.802	.8	0	.25	.05

a. Two-sided test.

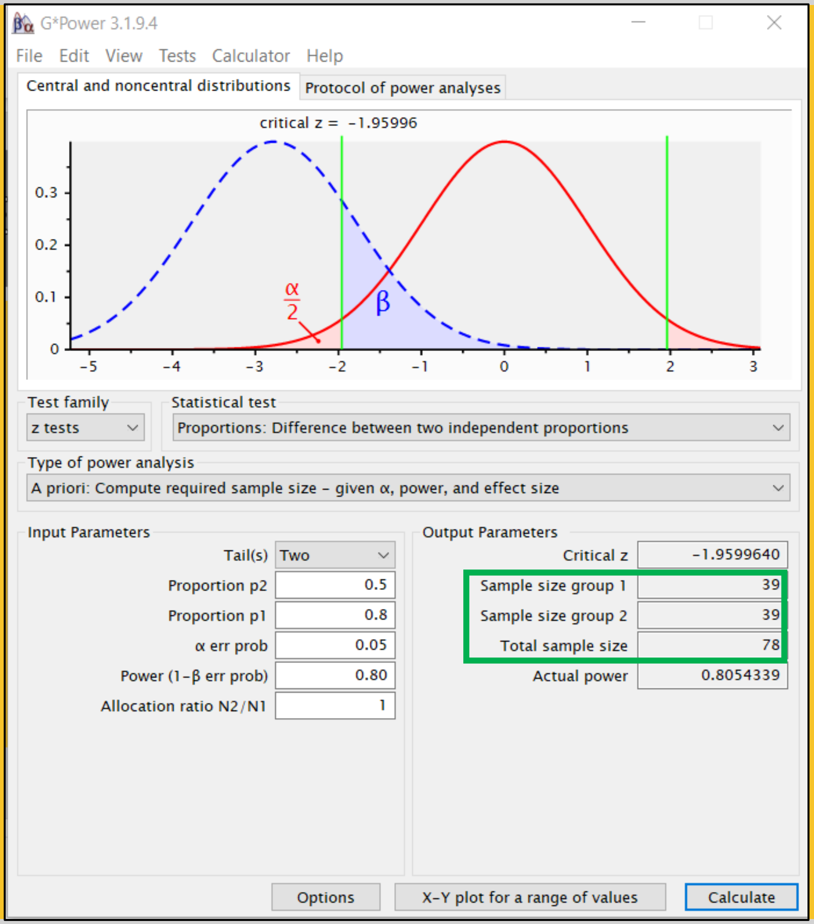
b. Based on Fisher's z-transformation and normal approximation with bias adjustment.

11. Proportion Example.

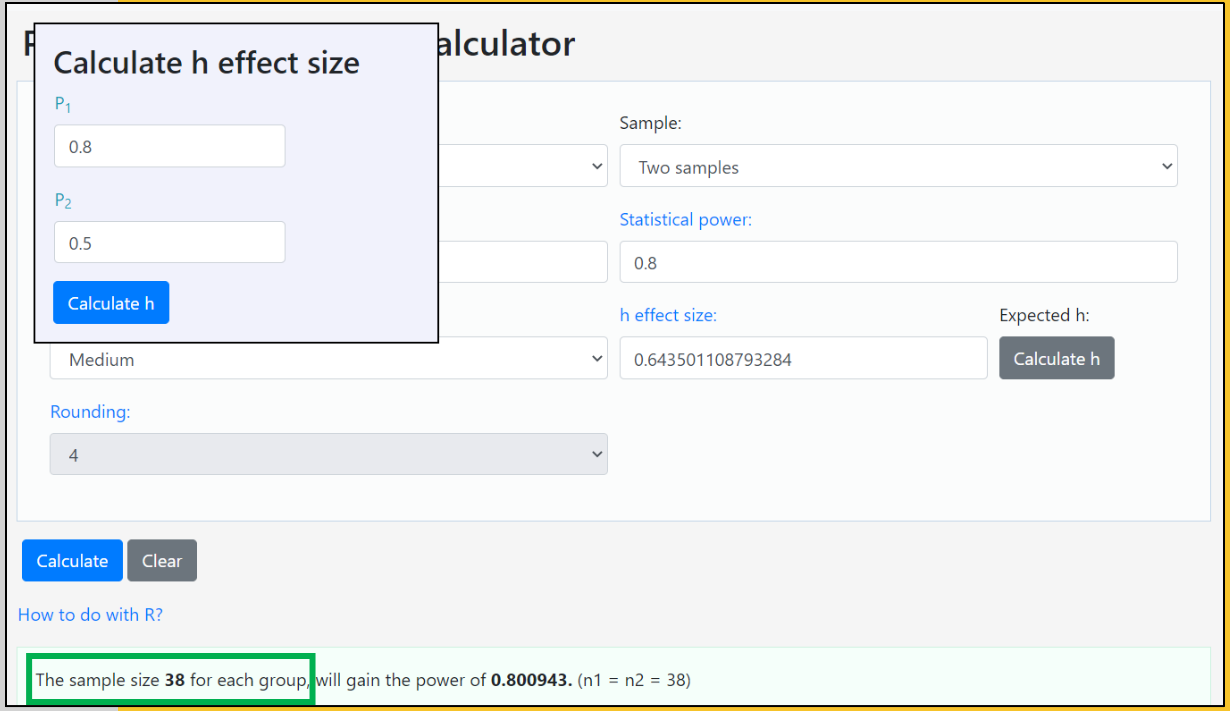
- **Data:**

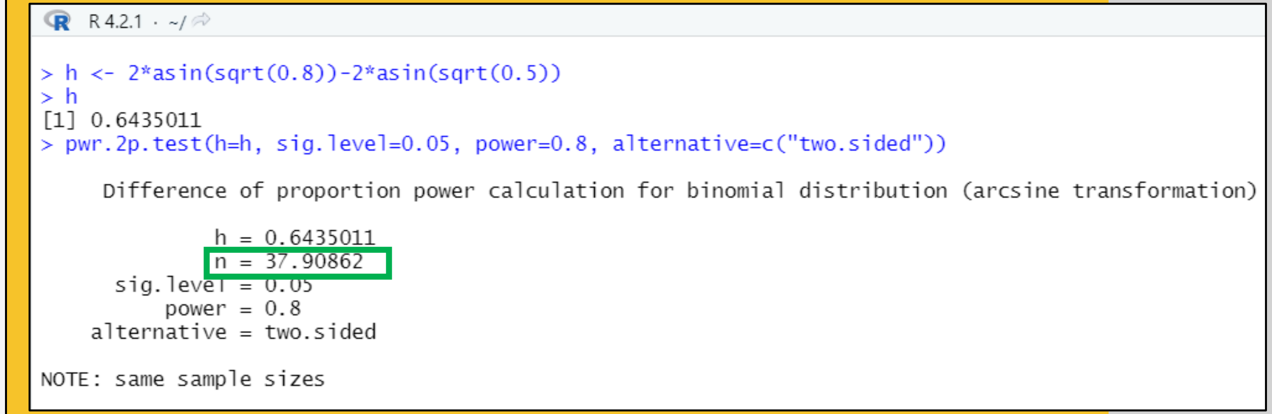
	Treat	Control
proportion	0.50	0.80

- **Method:**
  - G*Power
  - Online calculator
  - R
  - SPSS

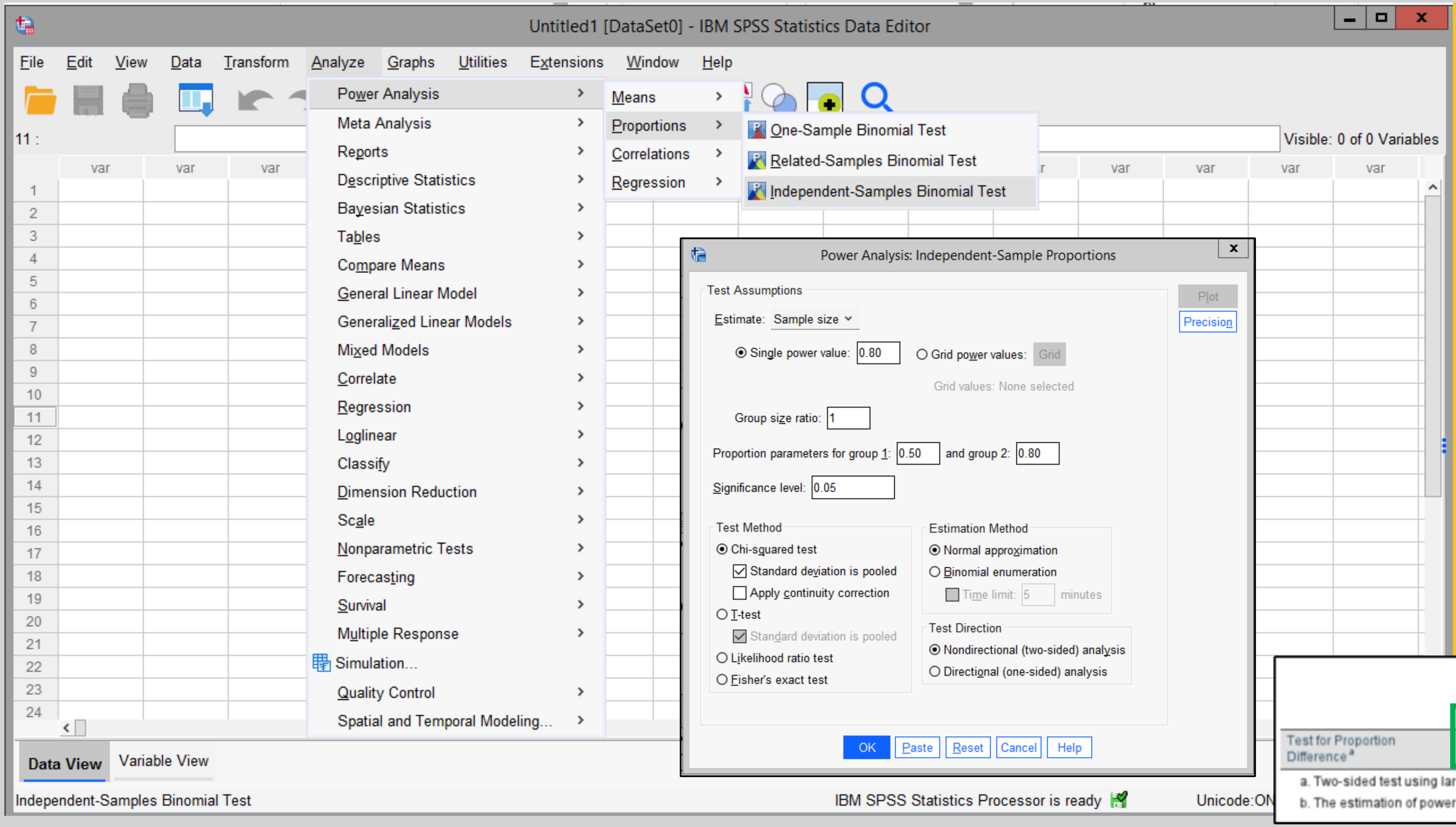


11. Proportion Example.





11. Proportion Example.



Power Analysis Table

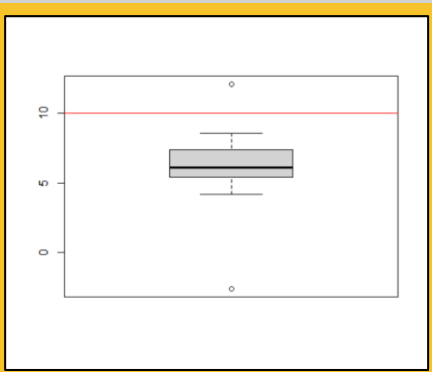
	N1	N2	Actual Power[b]	Test Assumptions: Power	Risk Difference	Risk Ratio	Odds Ratio	Sig.
Test for Proportion Difference[a]	39	39	.805	.8	-.300	.625	.250	.05

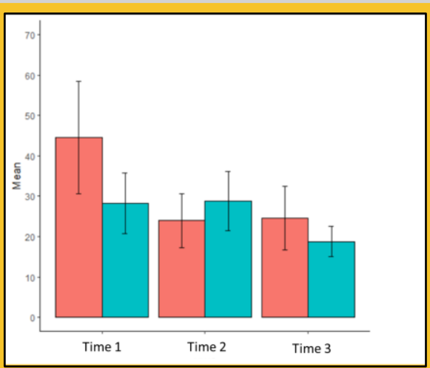
a. Two-sided test using large-sample approximation.

b. The estimation of power is based on the Pearson Chi-Square test and the pooled standard deviation.

12. Additional Designs.

- 1-sample or Paired T-test
- Repeated Measures
- Regression
- Chi-squared
- …





Expected	Observed
9	56
3	18
3	7
1	7

$$Y = \beta_0 + \beta_1 * X_1 + \beta_2 * X_2 + \epsilon$$

"A Power Analysis for every analysis"

13. Practical Limitations.

- Understanding what goes in.
- Simulation sanity check. (A)
- Realistic patient/sample number.
- Balance of art and science.

(A)

Mean A	**30**
Mean B	**22**
Std	**12**

d = (30-22) / 12
d = 0.667

```
> pwr.t.test(d=0.667, sig.level=0.05, power=0.8, type=c("two.sample"))

     Two-sample t test power calculation

              n = 36.27042
              d = 0.667
      sig.level = 0.05
          power = 0.8
    alternative = two.sided

NOTE: n is number in *each* group
```

```
> Treat<-c(rep("A",37), rep("B",37))
> Response<-round(c(rnorm(37, mean=30, sd=12), rnorm(37, mean=22, sd=12)),1)
> ss1 <-data.frame(Treat=Treat, Response=Response)
> t.test(Response~Treat, data=ss1)

        Welch Two Sample t-test

data:  Response by Treat
t = 2.9245, df = 69.914, p-value = 0.004647
alternative hypothesis: true difference in means between group A and group B is not equal to 0
95 percent confidence interval:
  2.942929 15.565179
sample estimates:
mean in group A mean in group B
       31.42432        22.17027
```

14. Where to Next?

- More resources
  - Power Analysis in G*Power (videos and slides)
  - Power Analysis in R (videos and slides)
  - Power Analysis in R: GLMMs (videos and slides)
  - Advanced Power Analysis: Into the Weeds (video and slides)
- How to practice
  - Design mock experiments
  - Calculate/guess effect size
  - Get comfortable using calculator/software
- Handy handouts [12]
  - Effect sizes for common designs

DACCOTA Statistical Resources

The Biostatistics, Epidemiology, and Research Design Core (BERDC) offers a variety of statistical resources from both our core and other biostatistical cores.

Effect Sizes for common designs

Statistical Test	Effect size	Equation	Rule of thumb for effect sizes
1 sample t-test	Cohen's d	$d = (\text{mean} - \text{constant}) / SD$	small=0.20, medium=0.50, large=0.80
2 sample t-test	Cohen's d	$d = (\text{mean}_1 - \text{mean}_2) / SD_{pooled}$	small=0.20, medium=0.50, large=0.80
Paired t-test	Cohen's d	$d = (\text{mean}_1 - \text{mean}_2) / SD_{pooled}$	small=0.20, medium=0.50, large=0.80
1-Way ANOVA	Eta squared Cohen's f	$\eta^2 = SS_{treatment} / SS_{total}$ $f = \text{sqrt}(\eta^2 / (1 - \eta^2))$	small=0.01, medium=0.05, large=0.14 small=0.10, medium=0.25, large=0.40
2-Way ANOVA	Eta squared Cohen's f	$\eta^2 = SS_{treatment} / SS_{total}$ $f = \text{sqrt}(\eta^2 / (1 - \eta^2))$	small=0.01, medium=0.06, large=0.14 small=0.10, medium=0.25, large=0.40
Repeated Measures ANOVA	Partial Eta squared Cohen's f	Partial $\eta^2 = SS_{effect} / (SS_{effect} + SS_{error})$ ...	small=0.01, medium=0.06, large=0.14 small=0.10, medium=0.25, large=0.40
1 proportion test	Cohen's h	$h = 2*\text{asin}(\text{sqrt}(prop_1)) - 2*\text{asin}(\text{sqrt}(prop_{const}))$	small=0.20, medium=0.50, large=0.80
2 proportions test	Cohen's h	$h = 2*\text{asin}(\text{sqrt}(prop_1)) - 2*\text{asin}(\text{sqrt}(prop_2))$	small=0.20, medium=0.50, large=0.80
Chi-squared test	Cohen's w	$w = \text{sqrt}(\sum (prop_{obs} - prop_{exp})^2 / prop_{exp})$	small=0.10, medium=0.30, large=0.50
Pearson Correlation	Correlation (R)	...	small=0.10, medium=0.30, large=0.50
Linear Regression (Entire Model)	F squared	$f^2 = R^2_{model} / (1 - R^2_{model})$	small=0.02, medium=0.15, large=0.35
Linear Regression (Ind. Predictor)	F squared	$f^2 = R^2_{increase} / (1 - R^2_{increase})$	small=0.10, medium=0.30, large=0.50

15. References and Acknowledgement.

DaCCoTA
DAKOTA COMMUNITY COLLABORATIVE ON TRANSLATIONAL ACTIVITY

[1] https://stats.oarc.ucla.edu/other/mult-pkg/seminars/intro-power/Others

[2] https://www.coursehero.com/file/p7g5rdk/Power-Analysis-Components-There-are-four-components-to-a-power-analysis-three/

[3] https://www.statology.org/effect-size/

[4] https://www.spss-tutorials.com/effect-size/#anova

[5] https://www.psychologie.hhu.de/arbeitsgruppen/allgemeine-psychologie-und-arbeitspsychologie/gpower

[6] http://powerandsamplesize.com/Calculators/

[7] https://www.gigacalculator.com/calculators/power-sample-size-calculator.php

[8] https://www.statskingdom.com/statistical-power-calculators.html

[9] https://sample-size.net

[10] https://www.statology.org/eta-squared/

[11] https://www.statology.org/partial-eta-squared/

[12]https://docs.google.com/spreadsheets/d/1dqbPqj3VfiHC3oZE4azLypiFOQaeoj9HQ8Z5yjOvybs/edit#gid=0

The DaCCoTA is supported by the National Institute of General Medical Sciences of the National Institutes of Health under Award Number U54GM128729.

For the labs that use the Biostatistics, Epidemiology, and Research Design Core in any way, including this Module, please acknowledge us for publications: "***Research reported in this publication was supported by DaCCoTA (the National Institute of General Medical Sciences of the National Institutes of Health under Award Number U54GM128729)***“