



Multivariate Analysis

Module II: Leaves and Trees

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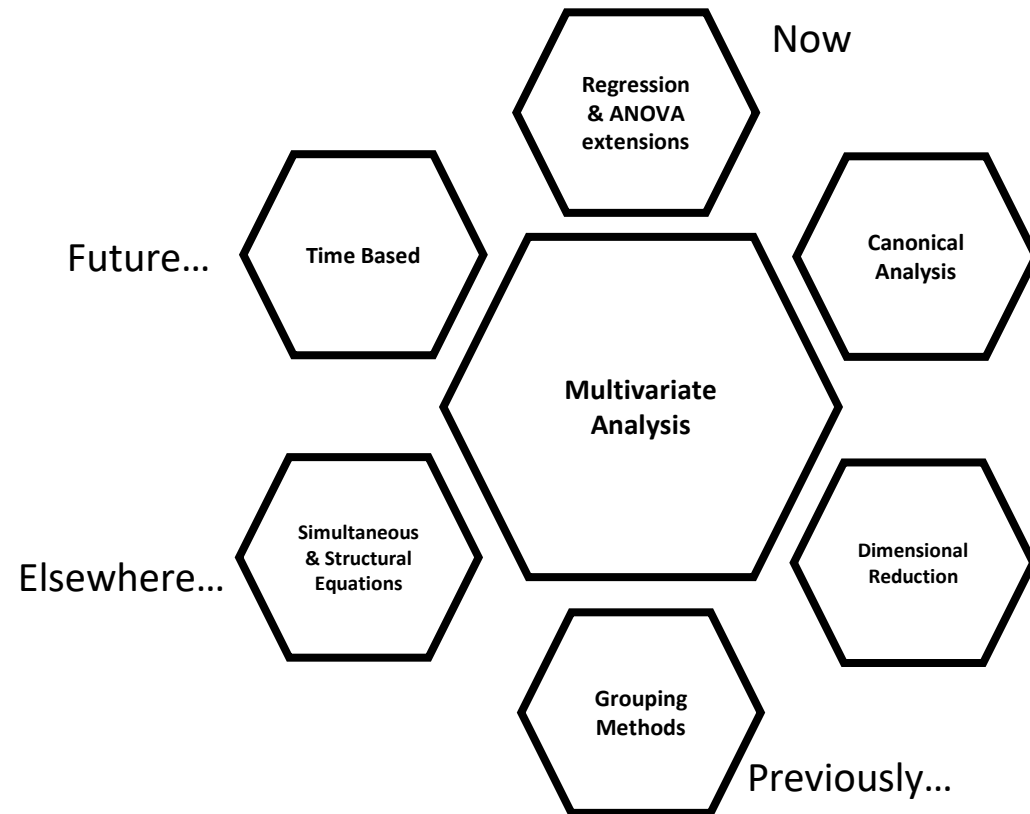
Introduction



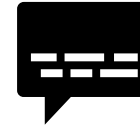
- Last time, we covered a broad overview of multivariate analysis

- Today we'll cover more details about specific multivariate methods

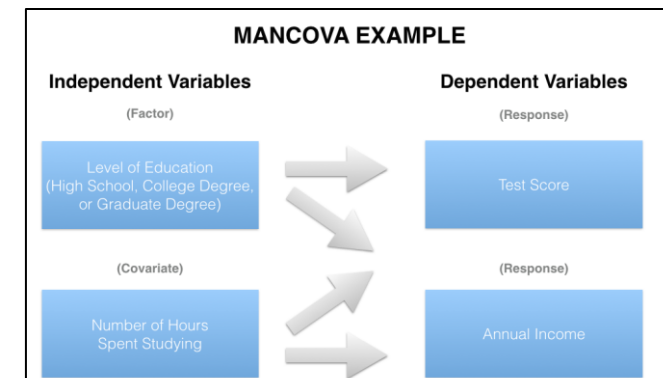
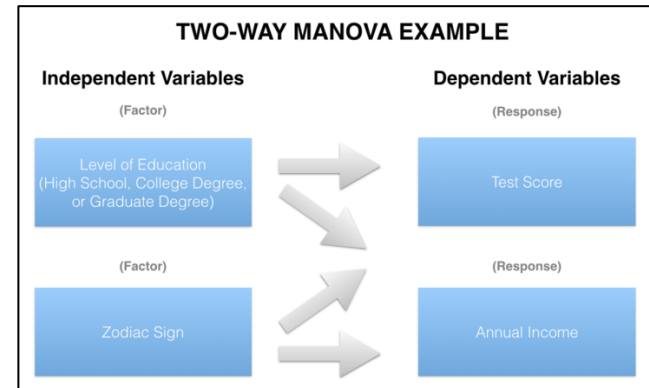
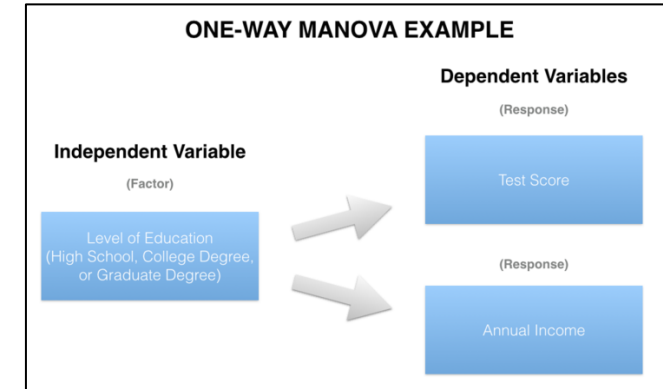
Multivariate:
multiple dependent variables or other more complicated structures such as ordination or non-linearity



Descriptions

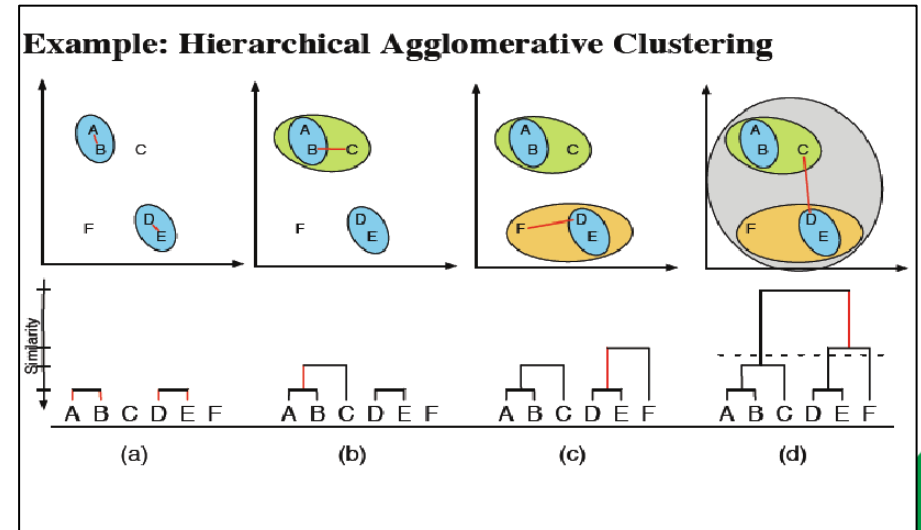
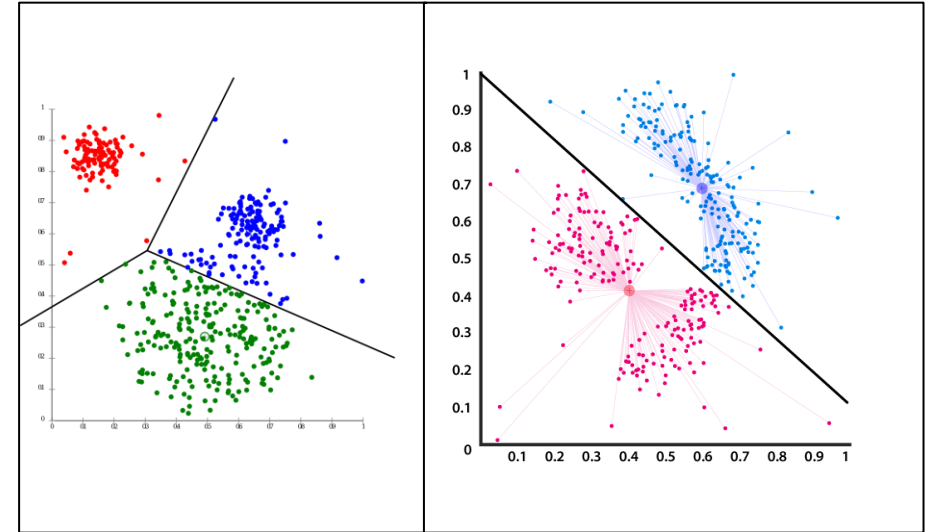


Name	Description
MANOVA	Multivariate analysis of variance. Used to test the statistical significance of the effect of one or more independent variables (categorical) on a set of two or more dependent variables
MANCOVA	Multivariate analysis of covariance. Same as MANOVA but after controlling for covariate(s).
Multivariate Regression	Used to test the statistical significance of the effect of one or more independent variables (numerical) on a set of two or more dependent variables
K-means clustering	Clustering method that partitions observations into k number of clusters, where each observation belongs to the cluster with the nearest mean. Also known as centroid-based clustering
Hierarchical clustering	Clustering method that separates observations based on a measure of similarity using a tree-based approach either from the bottom-up (agglomerative) or top-down (divisive)
Density-based clustering	Clustering method that connects areas where observations are high density and allows for arbitrary-shaped clusters.
Distribution-based clustering	Clustering method that assumes observations come from a certain distribution (such as Gaussian) and groups them with decreasing probability from the distribution's center.
Classification tree	Recursive partitioning decision tree in which target variables are categorical
Regression tree	Recursive partitioning decision tree in which target variables are numerical.



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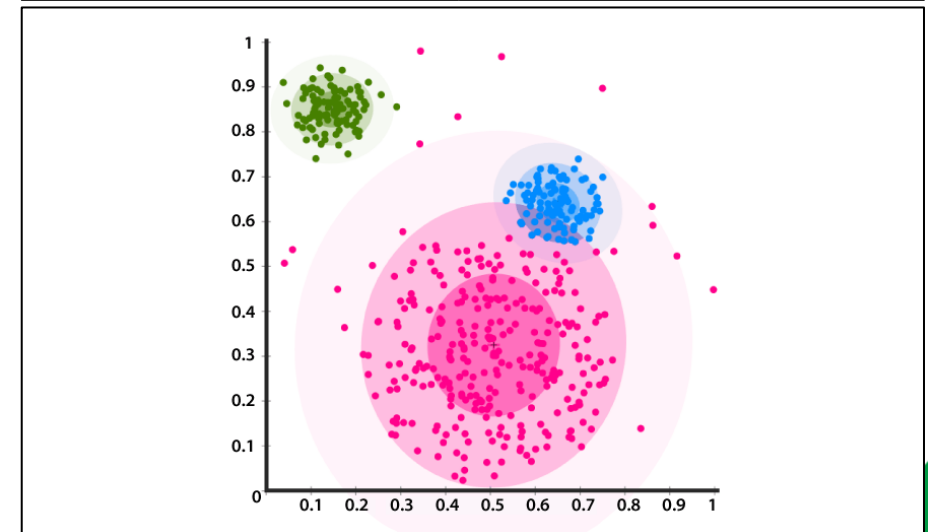
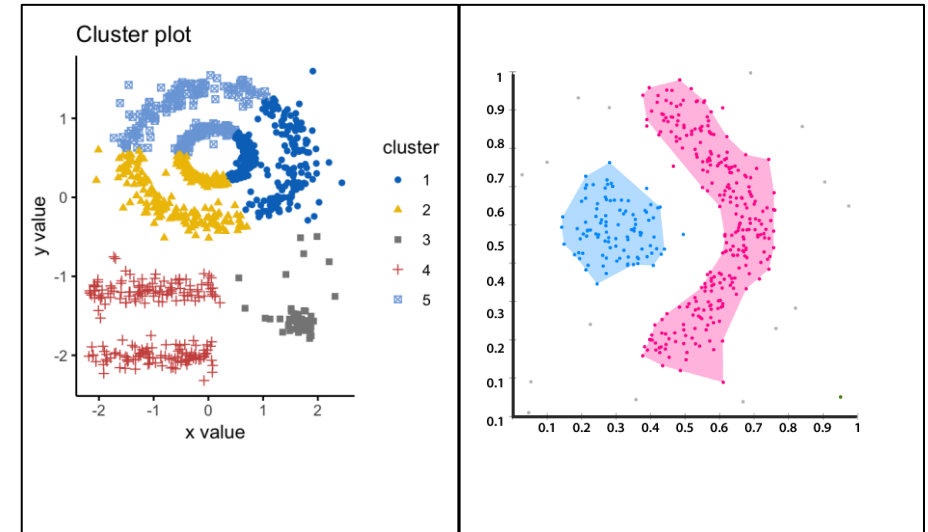
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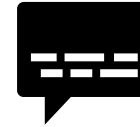
Descriptions



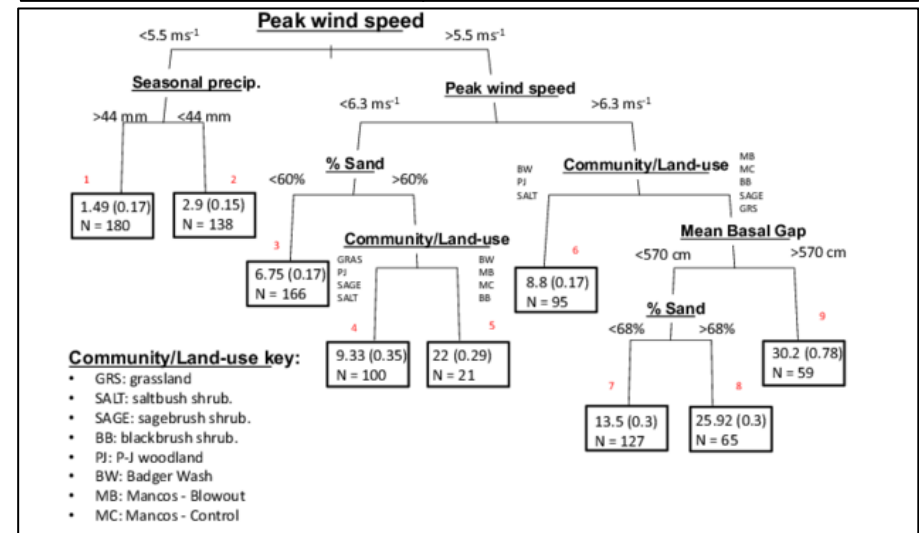
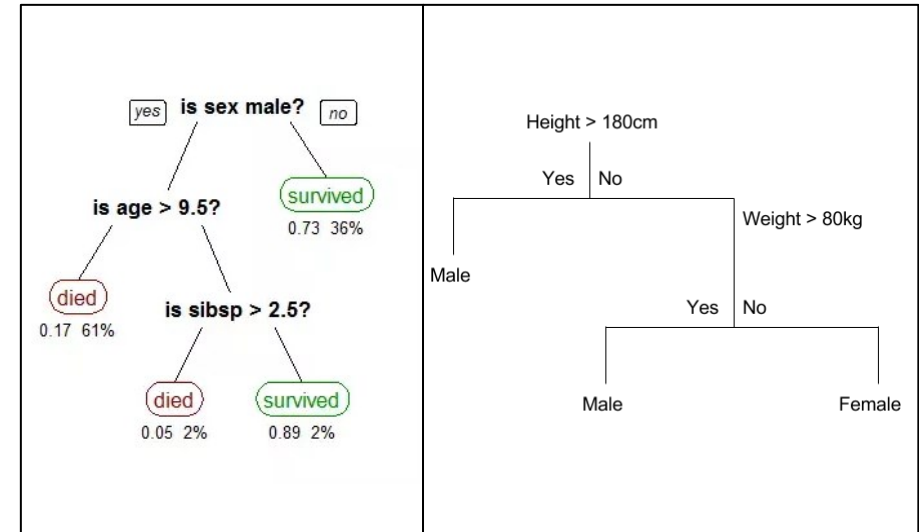
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Rationales



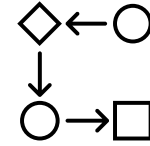
-  When and Why should you use multivariate analysis in general?
- Complex for a complex world
 - Don't use if you don't understand it
 - Don't use if a simpler method works

-  When should you use specific multivariate analysis methods?

-  What are the assumptions of multivariate analysis?

Name	Usage	Assumptions
1-Way MANOVA	Multiple numerical Y-variables (Responses), Single categorical X-variable (Factor)	<ul style="list-style-type: none"> • Independence of observations • Multivariate normality • Linearity (Y-vars) • No multicollinearity (X-vars) • Equality of variance • Equality of variance-covariance matrices
2-Way MANOVA	Multiple numerical Y-variables (Responses), Two categorical X-variables (Factors)	
1-WAY MANCOVA	Multiple numerical Y-variables (Responses), Single categorical X-variable of interest (Factor), Single numerical X-variable controlled for (Covariate)	
2-WAY MANCOVA	Multiple numerical Y-variables (Responses), Two categorical X-variable of interest (Factors), Single numerical X-variable controlled for (Covariate)	
Multivariate Regression	Multiple numerical Y-variables (Responses), numerical X-variables (Predictors)	<ul style="list-style-type: none"> • Same as MANOVA/MANCOVA except variance
K-means clustering	[Categorical Y-variable], Numerical X-variables, set number of clusters	<ul style="list-style-type: none"> • N/A
Hierarchical clustering	[Categorical Y-variable], Numerical X-variables, non-specified cluster number	
Distribution-based clustering	[Categorical Y-variable], Numerical X-variables, known distribution	
Density-based clustering	[Categorical Y-variable], Numerical X-variables, dataset with noise and/or outliers	
Classification tree	Categorical Y-variable (Outcome), Numerical or Categorical X-variables	<ul style="list-style-type: none"> • N/A
Regression tree	Continuous Y-variable (Outcome), Numerical or Categorical X-variables	

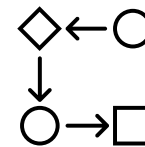
Step-by-step Example 1



MANOVA, MANCOVA, and multivariate regression using the mtcars dataset

- A. Is there a significant effect of transmission category (automatic/manual) and gear category (3/4/5) on MPG and quarter-mile time?
- B. Is there a significant effect of transmission category (automatic/manual) and gear category (3/4/5) on MPG and quarter-mile time, while accounting for weight?
- C. Is there a significant effect of displacement, gross horsepower, and rear axle ratio on MPG and quarter-mile time?

Step-by-step Example 1



Set-Up

#Intro stuff:

```
library(rstatix)
```

```
library(plyr)
```

```
library(tidyverse)
```

```
head(mtcars)
```

```
#outcomes (mpg and qsec); categorical  
predictors (am, gear); covariate (wt)
```

```
mtcars$am2 <-as.factor(mtcars$am)
```

```
mtcars$gear2 <-as.factor(mtcars$gear)
```

MANOVA/MANCOVA data exploration

#data visualization

```
par(mfrow=c(2,2))
```

```
plot(mtcars$mpg~mtcars$am2, col='orange')
```

```
plot(mtcars$mpg~mtcars$gear2, col='orange')
```

```
plot(mtcars$qsec~mtcars$am2, col='blue')
```

```
plot(mtcars$qsec~mtcars$gear2, col='blue')
```

```
plot(mtcars$mpg~mtcars$wt, col=mtcars$am2, pch=16)
```

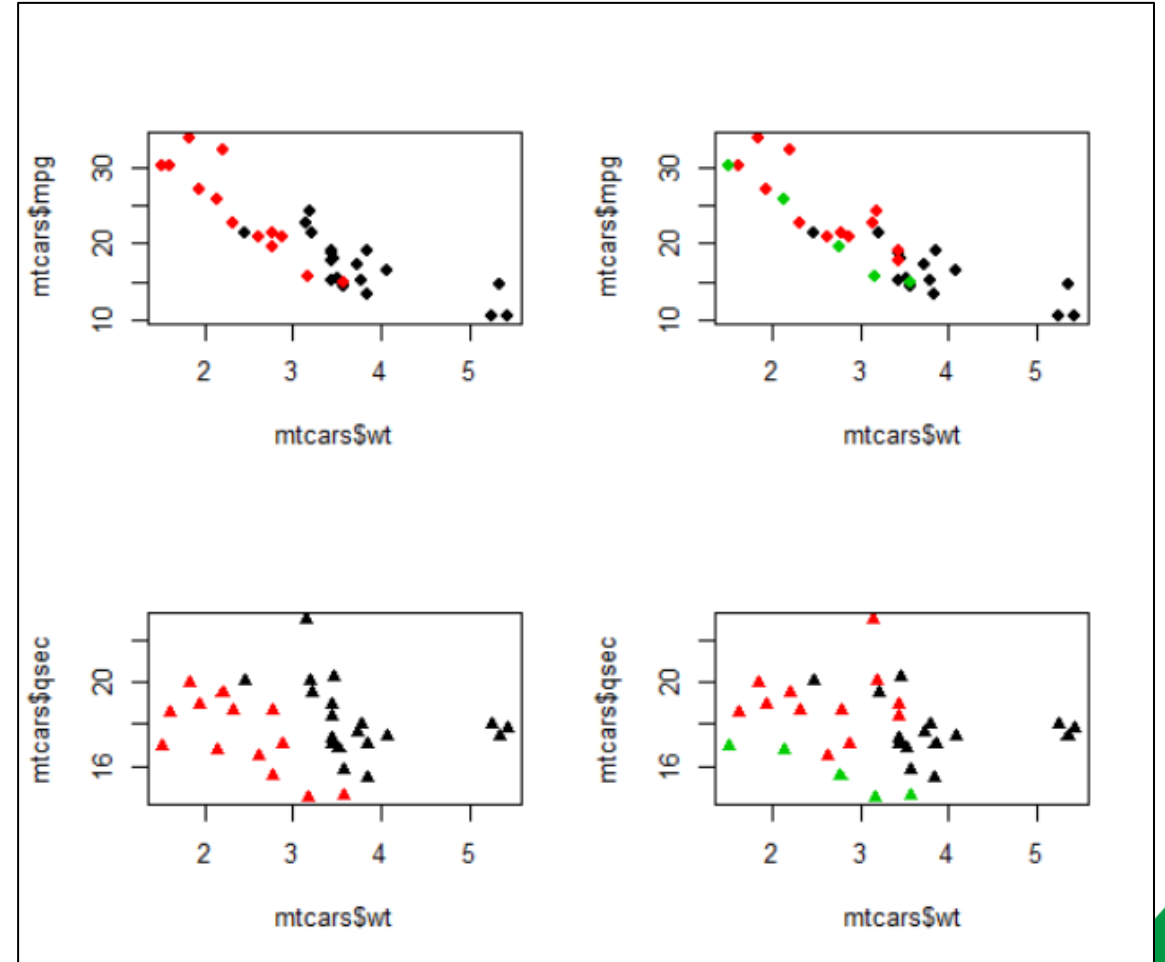
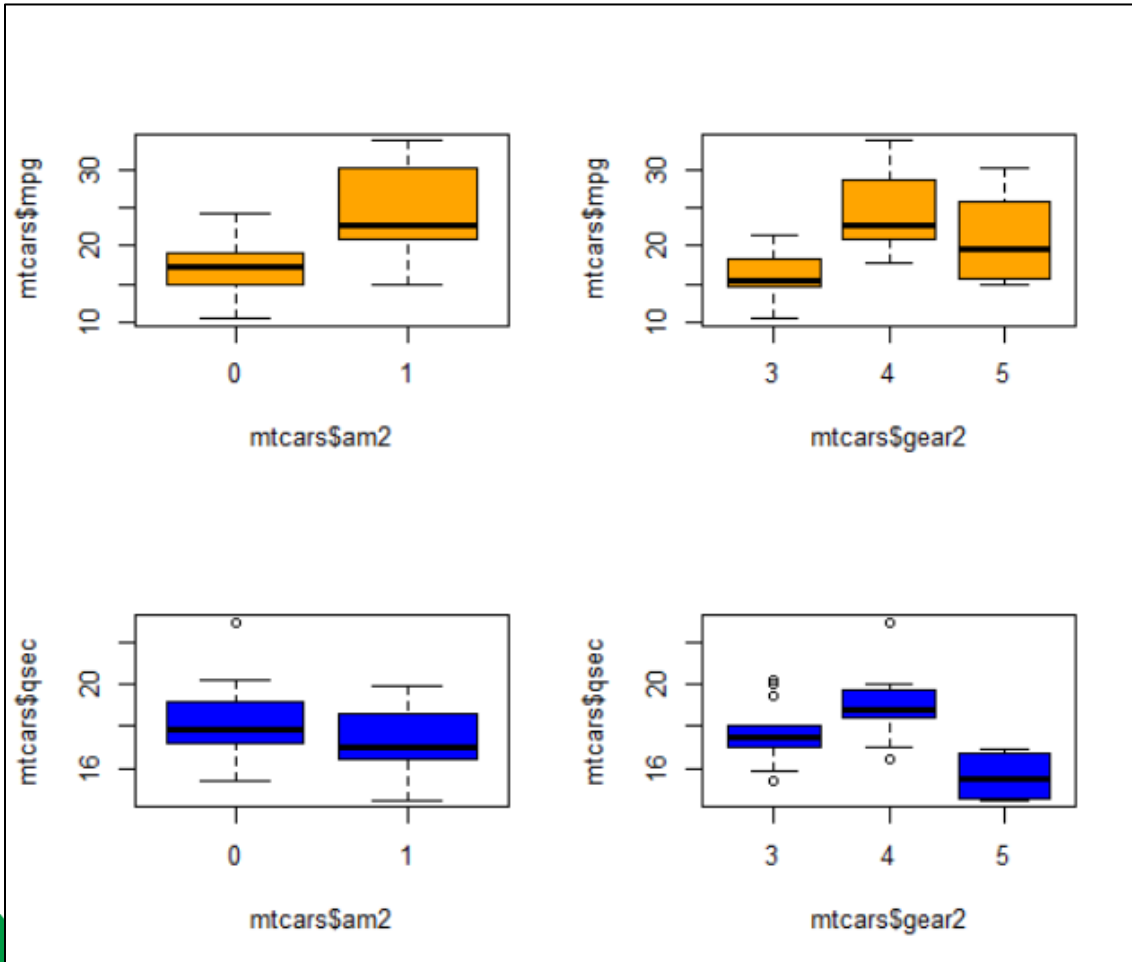
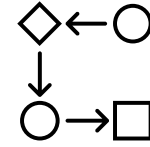
```
plot(mtcars$mpg~mtcars$wt, col=mtcars$gear2, pch=16)
```

```
plot(mtcars$qsec~mtcars$wt, col=mtcars$am2, pch=17)
```

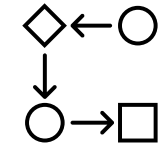
```
plot(mtcars$qsec~mtcars$wt, col=mtcars$gear2, pch=17)
```

```
par(mfrow=c(1,1))
```

Step-by-step Example 1



Step-by-step Example 1



Testing MANOVA/MANCOVA assumptions

#normality

```
hist(mtcars$mpg)
hist(mtcars$qsec) #good enough
```

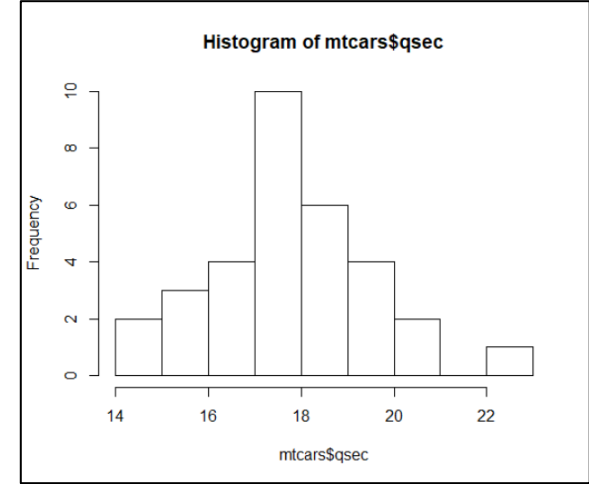
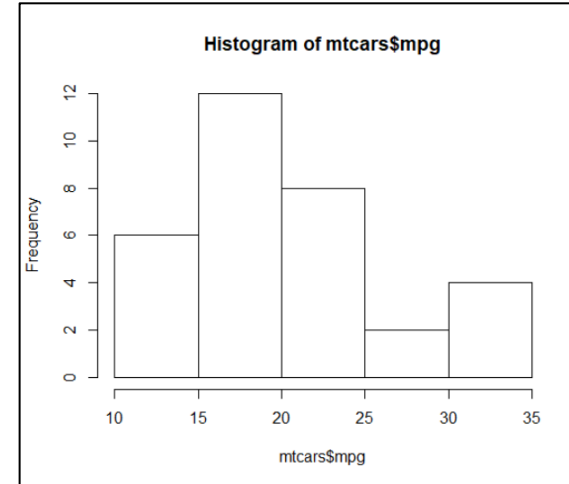
```
mtcars %>%
  select(mpg, qsec) %>%
  mshapiro_test() #good
```

#multicollinearity

```
cor.test(mtcars$mpg, mtcars$qsec) #good
```

#linearity

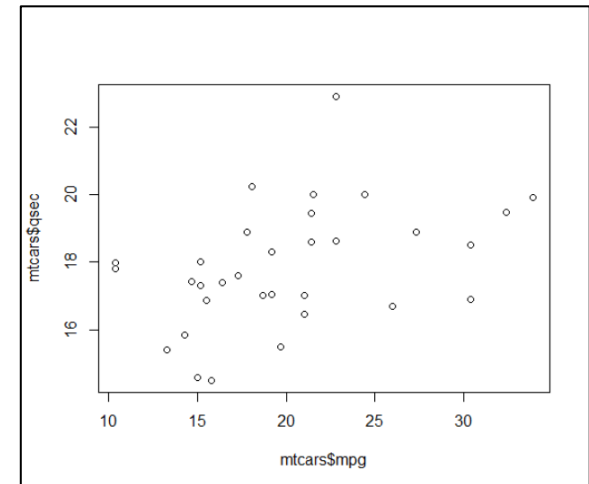
```
plot(mtcars$mpg, mtcars$qsec) #good
```



statistic	p.value
<dbl>	<dbl>
1	0.967 0.420

Pearson's product-moment correlation

data: mtcars\$mpg and mtcars\$qsec
 t = 2.5252, df = 30, p-value = 0.01708
 alternative hypothesis: true correlation is not equal to 0
 95 percent confidence interval:
 0.08195487 0.66961864
 sample estimates:
 cor
 0.418684



Step-by-step Example 1

Testing MANOVA/MANCOVA assumptions cont.

#homogeneity of variance and covariance

```
ldply(mtcars[,9:10],function(x) t(rbind(names(table(x)), table(x),
paste0(prop.table(table(x))*100, "%"))))
```

```
box_m(mtcars[, c("mpg", "qsec")], mtcars$am2) #good
box_m(mtcars[, c("mpg", "qsec")], mtcars$gear2) #significant (Pillai's)
```

```
mtcars %>%
gather(key = "variable", value = "value", mpg, qsec) %>%
group_by(variable) %>%
levene_test(value ~ am2) #good for mpg, not good for qsec
```

```
mtcars %>%
gather(key = "variable", value = "value", mpg, qsec) %>%
group_by(variable) %>%
levene_test(value ~ gear2) #not good for mpg, not good for qsec
```

	statistic	p.value	parameter	method
	<dbl>	<dbl>	<dbl>	<chr>
1	4.11	0.250	3	Box's M-test for Homogeneity of Covariance Matrices

	statistic	p.value	parameter	method
	<dbl>	<dbl>	<dbl>	<chr>
1	14.0	0.0298	6	Box's M-test for Homogeneity of Covariance Matrices

variable	df1	df2	statistic	p
<chr>	<int>	<int>	<dbl>	<dbl>
1 mpg	1	30	4.19	0.0496
2 qsec	1	30	0.322	0.575

variable	df1	df2	statistic	p
<chr>	<int>	<int>	<dbl>	<dbl>
1 mpg	2	29	1.49	0.242
2 qsec	2	29	0.0491	0.952

Step-by-step Example 1

MANOVA results

```
manova1 <- manova(cbind(mpg, qsec)~ am2 + gear2, data=mtcars)
summary(manova1)
summary.aov(manova1)
```

```
Df Pillai approx F num Df den Df Pr(>F)
am2 1 0.64944 25.0093 2 27 7.151e-07 ***
gear2 2 0.44423 3.9976 4 56 0.006369 **
Residuals 28
---
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Response mpg :
      Df Sum Sq Mean Sq F value Pr(>F)
am2    1 405.15 405.15 19.9021 0.0001208 ***
gear2  2 150.89 75.45 3.7062 0.0373294 *
Residuals 28 570.00 20.36
---
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Response qsec :
      Df Sum Sq Mean Sq F value Pr(>F)
am2    1 5.230 5.2301 2.7877 0.1061372
gear2  2 41.225 20.6125 10.9865 0.0003006 ***
Residuals 28 52.533 1.8762
---
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Step-by-step Example 1



MANCOVA results

```
mancova1 <-manova(cbind(mpg, qsec)~ am2 + gear2 + wt,
data=mtcars)
summary(mancova1)
summary.aov(mancova1)
```

	Df	Pillai approx	F num	Df den	Df	Pr(>F)
am2	1	0.76412	42.113	2	26	6.997e-09 ***
gear2	2	0.51596	4.694	4	54	0.002526 **
wt	1	0.59288	18.931	2	26	8.442e-06 ***

Residuals 27

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Response mpg :

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
am2	1	405.15	405.15	45.9872	2.778e-07 ***
gear2	2	150.89	75.45	8.5637	0.001318 **
wt	1	332.13	332.13	37.6990	1.464e-06 ***

Residuals 27 237.87 8.81

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

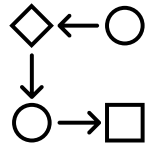
Response qsec :

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
am2	1	5.230	5.2301	3.1824	0.0856791 .
gear2	2	41.225	20.6125	12.5422	0.0001406 ***
wt	1	8.160	8.1598	4.9650	0.0343877 *

Residuals 27 44.373 1.6435

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Step-by-step Example 1



Multivariate Regression data exploration

```
#mpg
mtcars_mpg <- data.frame(mpg=mtcars$mpg,
disp=mtcars$disp, hp=mtcars$hp, wt=mtcars$wt)
pairs(mtcars_mpg, col="blue")
```

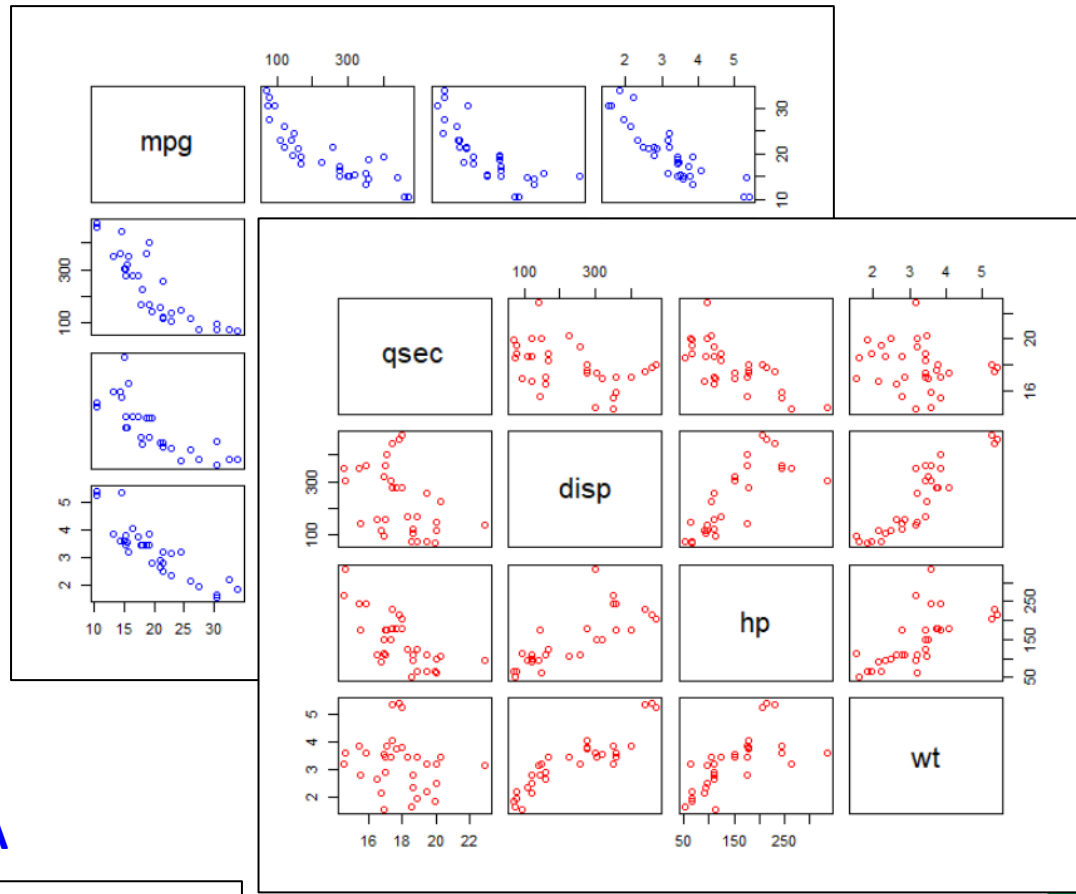
```
#qsec
mtcars_qsec <- data.frame(qsec=mtcars$qsec,
disp=mtcars$disp, hp=mtcars$hp, wt=mtcars$wt)
pairs(mtcars_qsec, col="red")
```

Multivariate Regression assumptions

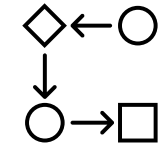
```
#normality and linearity
#good, based on previously done with MANOVA/MANCOVA
```

```
#multicollinearity the X-variables
cor(mtcars[,4:6])
```

	hp	drat	wt
hp	1.0000000	-0.4487591	0.6587479
drat	-0.4487591	1.0000000	-0.7124406
wt	0.6587479	-0.7124406	1.0000000



Step-by-step Example 1



Multivariate Regression results

```
mvreg1 <-lm(cbind(mpg,qsec) ~ disp + hp + wt, data=mtcars)
summary(mvreg1)
```

<p>Response mpg :</p> <p>Call: lm(formula = mpg ~ disp + hp + wt, data = mtcars)</p> <p>Residuals: Min 1Q Median 3Q Max -3.891 -1.640 -0.172 1.061 5.861</p> <p>Coefficients: Estimate Std. Error t value Pr(> t) (Intercept) 37.105505 2.110815 17.579 < 2e-16 *** disp -0.000937 0.010350 -0.091 0.92851 hp -0.031157 0.011436 -2.724 0.01097 * wt -3.800891 1.066191 -3.565 0.00133 **</p> <p>--- Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1</p> <p>Residual standard error: 2.639 on 28 degrees of freedom Multiple R-squared: 0.8268, Adjusted R-squared: 0.8083 F-statistic: 44.57 on 3 and 28 DF, p-value: 8.65e-11</p>	<p>Response qsec :</p> <p>Call: lm(formula = qsec ~ disp + hp + wt, data = mtcars)</p> <p>Residuals: Min 1Q Median 3Q Max -1.8121 -0.3125 -0.0245 0.3544 3.3693</p> <p>Coefficients: Estimate Std. Error t value Pr(> t) (Intercept) 17.965050 0.849663 21.144 < 2e-16 *** disp -0.006622 0.004166 -1.590 0.12317 hp -0.022953 0.004603 -4.986 2.88e-05 *** wt 1.485283 0.429172 3.461 0.00175 **</p> <p>--- Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1</p> <p>Residual standard error: 1.062 on 28 degrees of freedom Multiple R-squared: 0.6808, Adjusted R-squared: 0.6466 F-statistic: 19.91 on 3 and 28 DF, p-value: 4.134e-07</p>
--	---

Assessment 1



qualtrics^{XM}



https://und.qualtrics.com/jfe/form/SV_d7ozvbwOekrBahU

Step-by-step Example 2

Clustering and recursive partitioning using the mtcars dataset

- A. Can cars be clustering into groups by using the car characteristic variables?
- B. Can car MPG or engine type (V-shaped or straight) be predicted using car characteristic variables?

Step-by-step Example 2

Setup

#Intro stuff:

```
library(cluster)
```

```
library(factoextra)
```

```
library(dbSCAN)
```

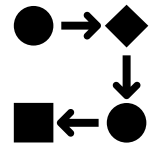
```
library(mclust)
```

```
head(mtcars)
```

```
mtcars2 <- mtcars[,1:7]
```

```
head(mtcars2)
```

Step-by-step Example 2



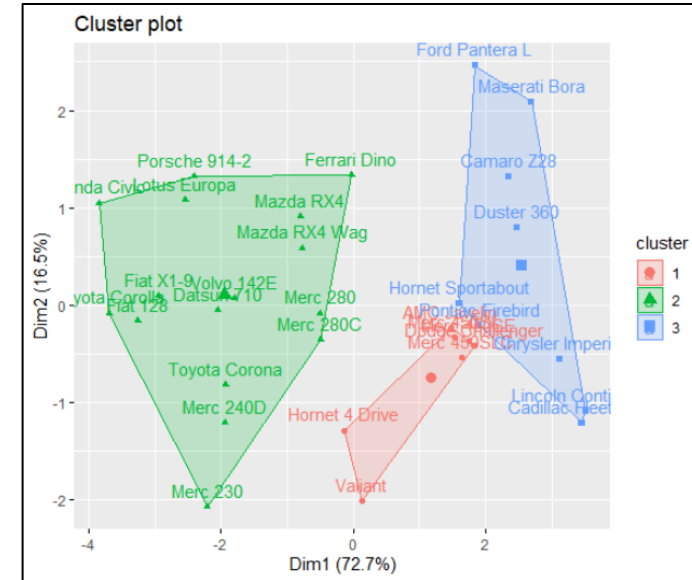
K-means clustering

```
kmeans1 <- kmeans(mtcars2, centers=2, nstart=100)
str(kmeans1)
```

```
fviz_cluster(kmeans1, data=mtcars2)
```

```
kmeans2 <- kmeans(mtcars2, centers=3, nstart=100)
```

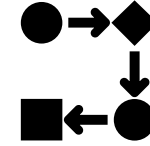
```
fviz_cluster(kmeans2, data=mtcars2)
```



List of 9

```
$ cluster : Named int [1:32] 2 2 2 2 1 2 1 2 2 2 ...
..- attr(*, "names")= chr [1:32] "Mazda RX4" "Mazda RX4 Wag" "Datsun 710" "Hornet 4 Drive" ...
$ centers : num [1:2, 1:7] 15.1 23.97 8 4.78 353.1 ...
..- attr(*, "dimnames")=List of 2
...$ : chr [1:2] "1" "2"
...$ : chr [1:7] "mpg" "cyl" "disp" "hp" ...
$ totss : num 623274
$ withinss : num [1:2] 93604 58870
$ tot.withinss: num 152473
$ betweenss : num 470801
$ size : int [1:2] 14 18
$ iter : int 1
$ ifault : int 0
- attr(*, "class")= chr "kmeans"
```

Step-by-step Example 2

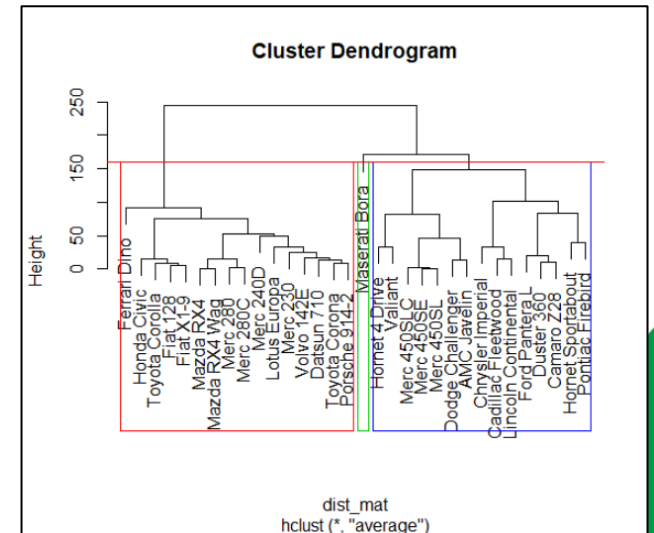
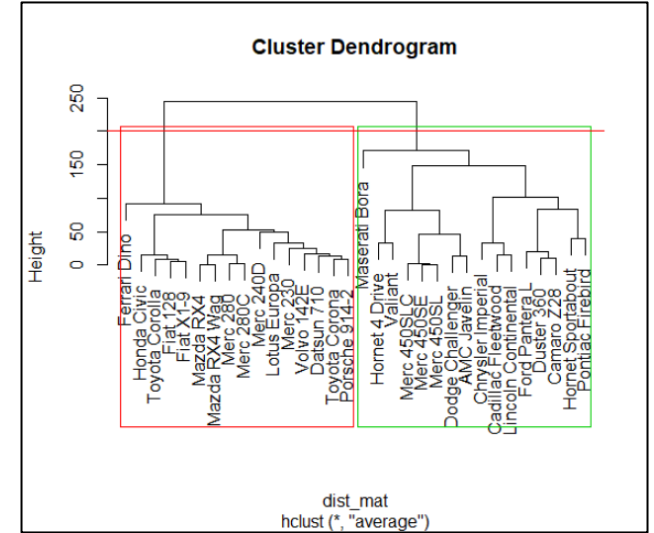
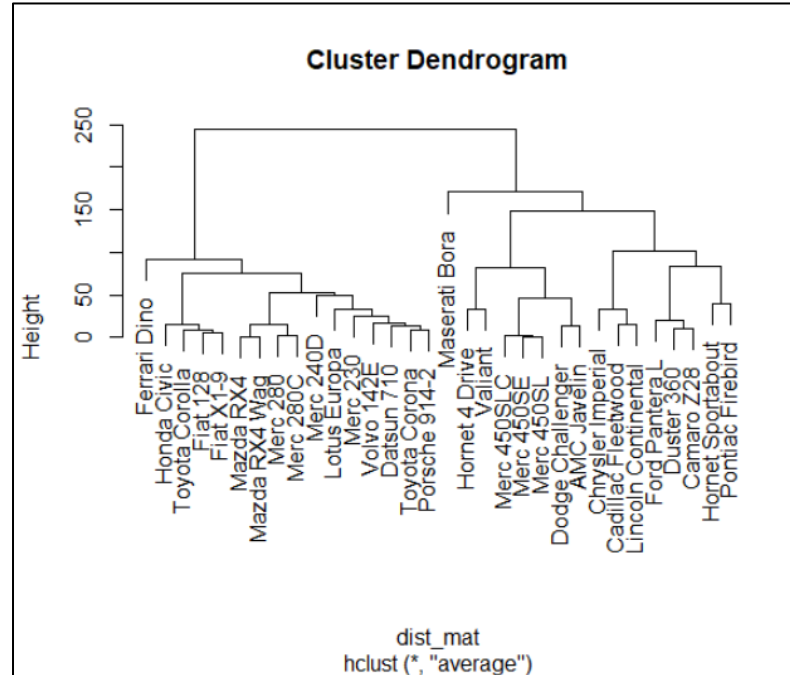


Hierarchical clustering

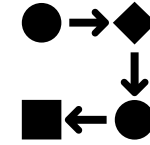
```
dist_mat <- dist(mtcars2, method="euclidean")
hclust1 <- hclust(dist_mat, method='average')
plot(hclust1)
```

```
plot(hclust1)
rect.hclust(hclust1, k=2, border=2:6)
abline(h=200, col="red")
```

```
plot(hclust1)
rect.hclust(hclust1, k=3, border=2:6)
abline(h=160, col="red")
```



Step-by-step Example 2



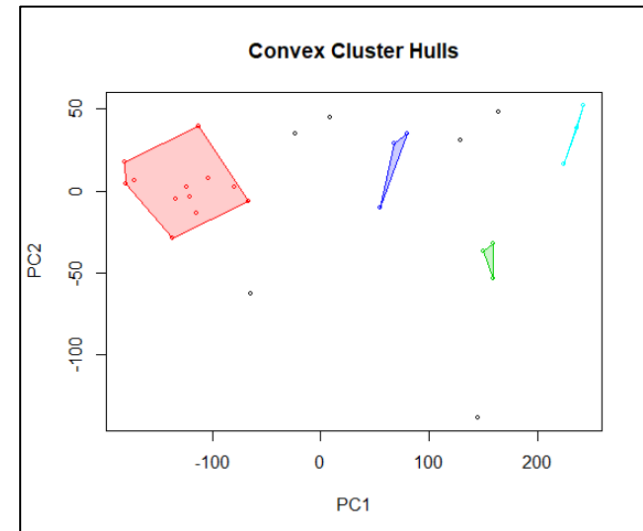
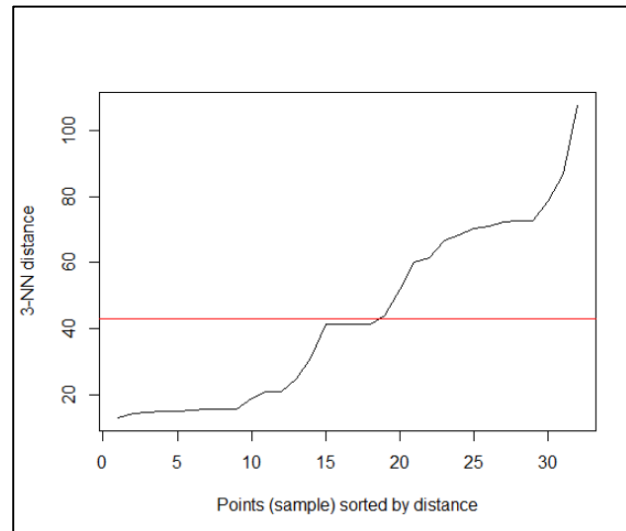
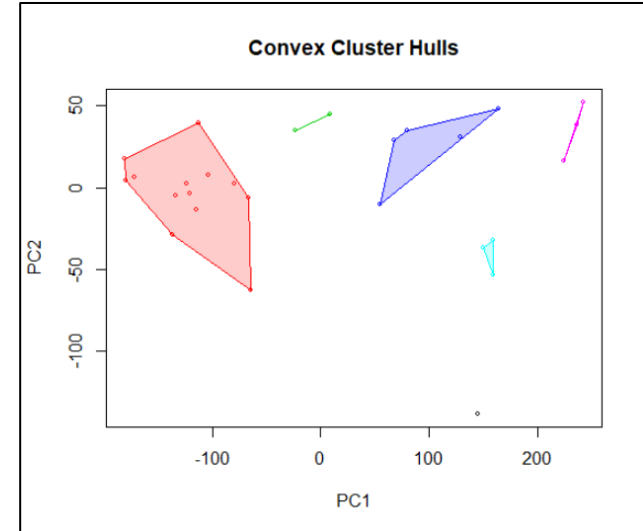
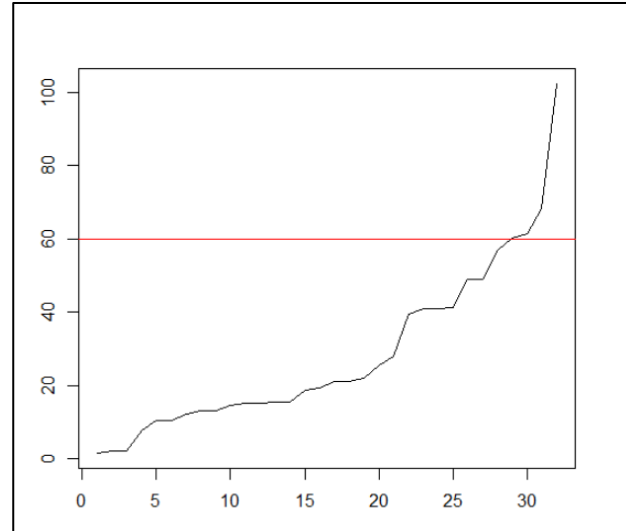
Density-based clustering

```
kNNdistplot(mtcars2, k=2)  
abline(h=60, col="red")
```

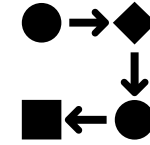
```
dbclust1 <- dbscan(mtcars2, 60, 2)  
hullplot(mtcars2, dbclust1$cluster)
```

```
kNNdistplot(mtcars2, k=3)  
abline(h=43, col="red")
```

```
dbclust2 <- dbscan(mtcars2, 43, 3)  
hullplot(mtcars2, dbclust2$cluster)
```



Step-by-step Example 2



Distribution-based clustering

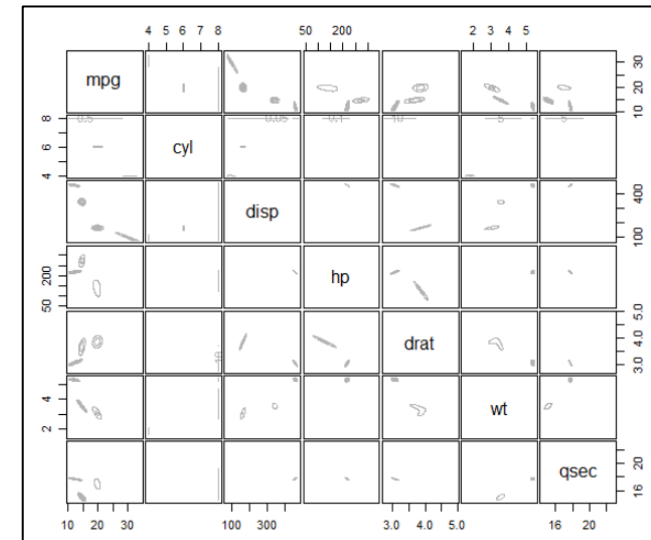
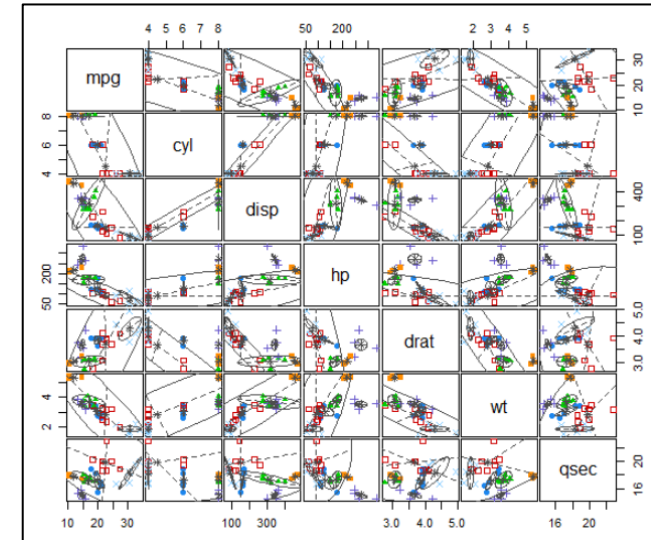
```
mclust1 <- Mclust(mtcars2)
mclust1$modelName
mclust1$G
plot(mclust1, what=c('classification'))
plot(mclust1, "density")
```

```
[1] "VEV"
[1] 6
```

```
mtcars3 <- mtcars2[,1:3]
```

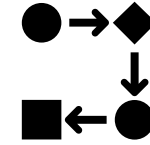
```
mclust2 <- Mclust(mtcars3)
mclust2$modelName
mclust2$G
plot(mclust2, what=c('classification'))
plot(mclust2, "density")
```

```
mclust3 <- Mclust(mtcars3, 2)
mclust3$modelName
mclust3$G
plot(mclust3, what=c('classification'))
plot(mclust3, "density")
```



<https://en.proft.me/2017/02/1/model-based-clustering-r/>

Step-by-step Example 2



Distribution-based clustering

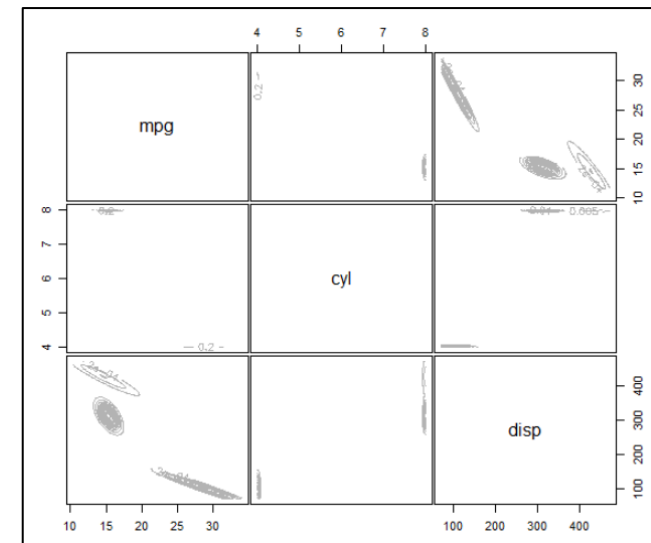
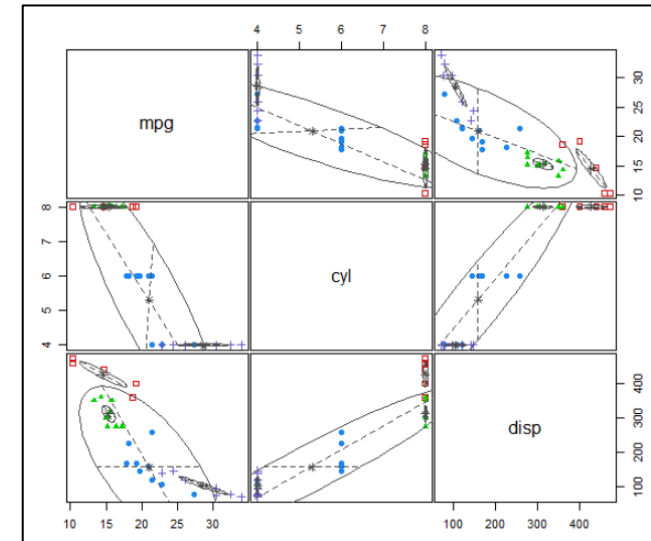
```
mclust1 <- Mclust(mtcars2)
mclust1$modelName
mclust1$G
plot(mclust1, what=c('classification'))
plot(mclust1, "density")
```

```
mtcars3 <- mtcars2[,1:3]
```

```
mclust2 <- Mclust(mtcars3)
mclust2$modelName
mclust2$G
plot(mclust2, what=c('classification'))
plot(mclust2, "density")
```

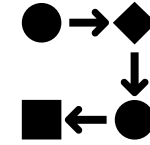
```
mclust3 <- Mclust(mtcars3, 2)
mclust3$modelName
mclust3$G
plot(mclust3, what=c('classification'))
plot(mclust3, "density")
```

```
[1] "VEV"
[1] 4
```



<https://en.proft.me/2017/02/1/model-based-clustering-r/>

Step-by-step Example 2



Distribution-based clustering

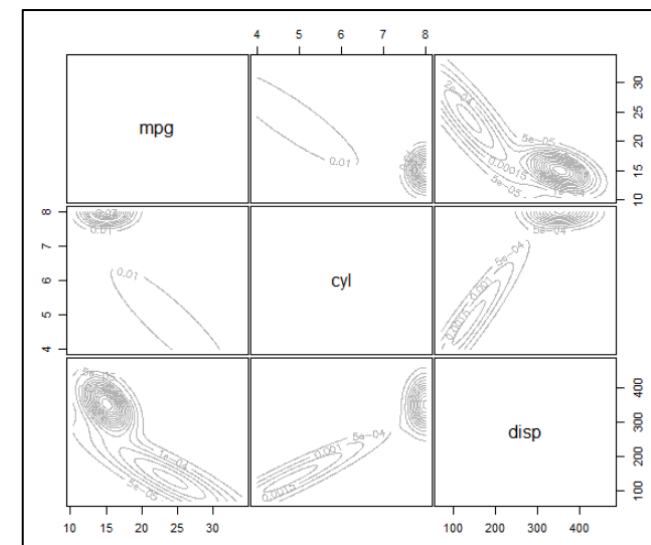
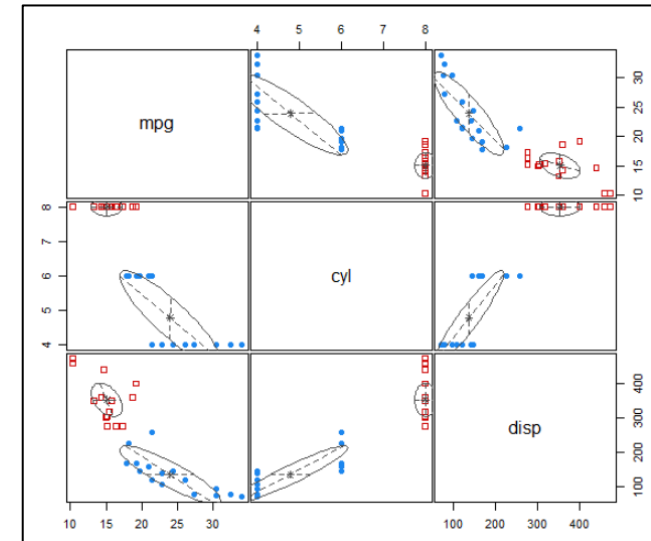
```
mclust1 <- Mclust(mtcars2)
mclust1$modelName
mclust1$G
plot(mclust1, what=c('classification'))
plot(mclust1, "density")
```

```
mtcars3 <- mtcars2[,1:3]
```

```
mclust2 <- Mclust(mtcars3)
mclust2$modelName
mclust2$G
plot(mclust2, what=c('classification'))
plot(mclust2, "density")
```

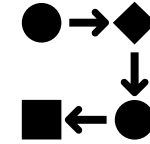
```
mclust3 <- Mclust(mtcars3, 2)
mclust3$modelName
mclust3$G
plot(mclust3, what=c('classification'))
plot(mclust3, "density")
```

```
[1] "VEV"
[1] 2
```



<https://en.proft.me/2017/02/1/model-based-clustering-r/>

Step-by-step Example 2



Regression Trees

<https://www.statology.org/classification-and-regression-trees-in-r/>

```
library(rpart)
library(rpart.plot)
head(mtcars)
```

```
tree1 <- rpart(mpg ~ disp + hp + drat + wt + qsec, data=mtcars,
control=rpart.control(cp=0.0001))
```

```
printcp(tree1)
prp(tree1)
```

```
best <- tree1$cptable[which.min(tree1$cptable[, "xerror"]), "CP"]
pruned_tree1 <- prune(tree1, cp=best)
prp(pruned_tree1, faclen=0, extra=1, roundint=F, digits=4)
```

```
#predict
tree2 <- rpart(mpg~disp+wt,data=mtcars, control=rpart.control(cp=0.0001))
prp(tree2)
new_car <- data.frame(wt=3, disp=300)
predict(tree2, newdata=new_car)
```

15.1

Regression tree:

```
rpart(formula = mpg ~ disp + hp + drat + wt + qsec, data = mtcars,
control = rpart.control(cp = 1e-04))
```

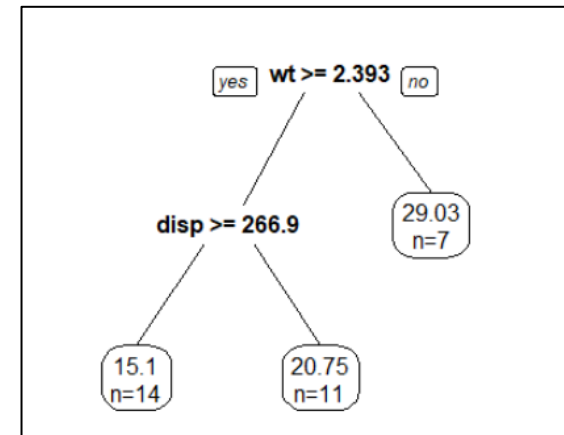
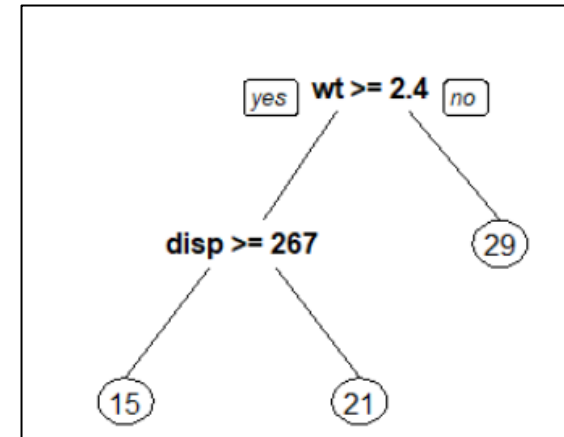
Variables actually used in tree construction:

[1] disp wt

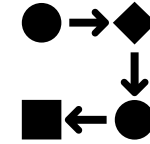
Root node error: 1126/32 = 35.189

n= 32

	CP	nsplit	rel error	xerror	xstd
1	0.63566	0	1.00000	1.09142	0.25931
2	0.17491	1	0.36434	0.74268	0.17098
3	0.00010	2	0.18943	0.59461	0.12028



Step-by-step Example 2



Classification Trees

```
mtcars$vs2 <-as.factor(mtcars$vs)
mtcars$am2 <-as.factor(mtcars$am)
mtcars$gear2 <-as.factor(mtcars$gear)
mtcars$carb2 <-as.factor(mtcars$carb)
str(mtcars)
```

```
tree3 <-rpart(vs2 ~ mpg + cyl + disp + hp + drat +wt +qsec +am2
+gear2 +carb2, data=mtcars, control=rpart.control(cp=0.0001))
```

```
printcp(tree3)
prp(tree3,faclen=0, extra=1, roundint=F, digits=4)
```

```
#less predictive
tree4 <-rpart(vs2 ~ drat +am2 +carb2, data=mtcars,
control=rpart.control(cp=0.0001))
```

```
printcp(tree4)
prp(tree4,faclen=0, extra=1, roundint=F, digits=4)
```

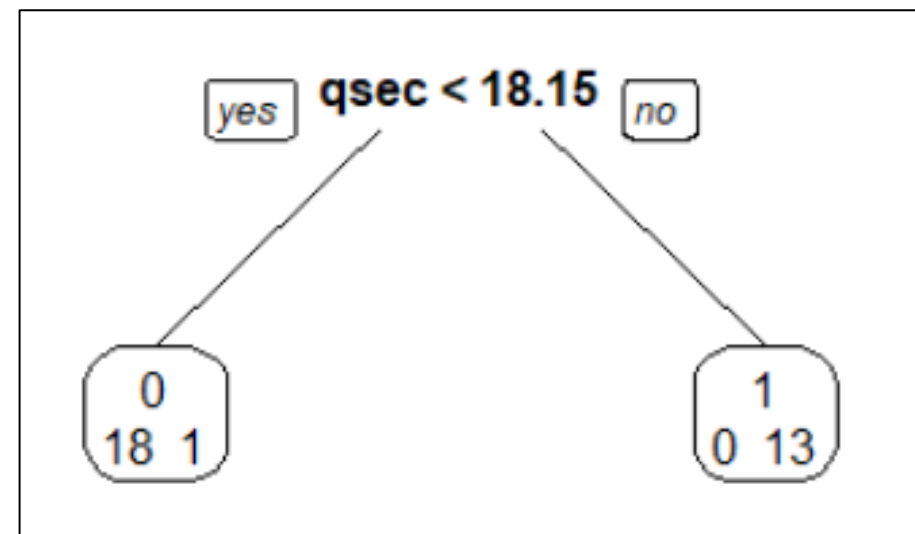
Classification tree:
rpart(formula = vs2 ~ mpg + cyl + disp + hp + drat + wt + qsec +
am2 + gear2 + carb2, data = mtcars, control = rpart.control(cp = 1e-04))

Variables actually used in tree construction:
[1] qsec

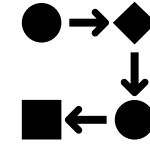
Root node error: 14/32 = 0.4375

n= 32

CP	nsplit	rel error	xerror	xstd	
1	0.92857	0	1.000000	1.000000	0.200446
2	0.00010	1	0.071429	0.071429	0.070304



Step-by-step Example 2



Classification Trees

```
mtcars$vs2 <-as.factor(mtcars$vs)
mtcars$am2 <-as.factor(mtcars$am)
mtcars$gear2 <-as.factor(mtcars$gear)
mtcars$carb2 <-as.factor(mtcars$carb)
str(mtcars)
```

```
tree3 <-rpart(vs2 ~ mpg + cyl + disp + hp + drat +wt +qsec +am2
+gear2 +carb2, data=mtcars, control=rpart.control(cp=0.0001))
```

```
printcp(tree3)
prp(tree3,faclen=0, extra=1, roundint=F, digits=4)
```

```
#less predictive
tree4 <-rpart(vs2 ~ drat +am2 +carb2, data=mtcars,
control=rpart.control(cp=0.0001))
```

```
printcp(tree4)
prp(tree4,faclen=0, extra=1, roundint=F, digits=4)
```

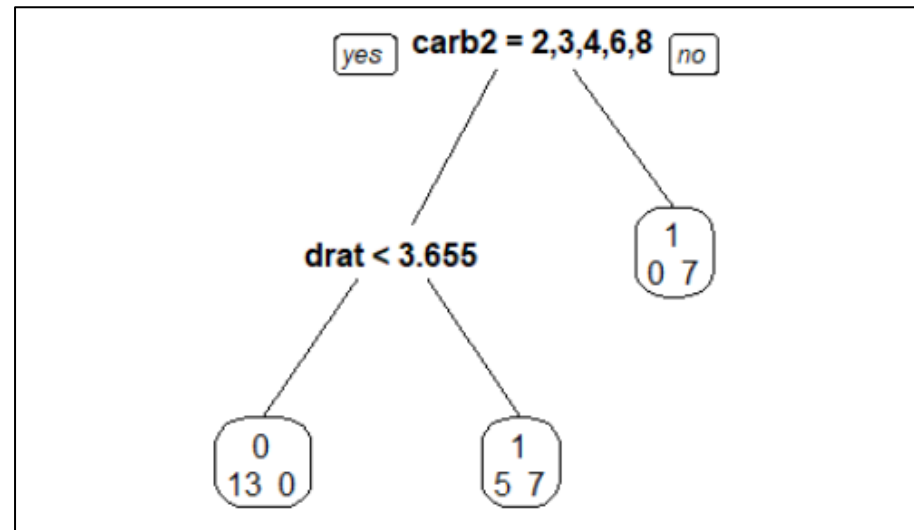
Classification tree:
 rpart(formula = vs2 ~ drat + am2 + carb2, data = mtcars, control = rpart.control(cp = 1e-04))

Variables actually used in tree construction:
 [1] carb2 drat

Root node error: 14/32 = 0.4375

n= 32

	CP	nsplit	rel error	xerror	xstd
1	0.50000	0	1.00000	1.00000	0.20045
2	0.14286	1	0.50000	0.92857	0.19845
3	0.00010	2	0.35714	0.92857	0.19845



Assessment 2



qualtrics^{XM}



https://und.qualtrics.com/jfe/form/SV_6ustS4Q8hjLwLqK

Caveats and Concerns



MANOVA and MANCOVA

- A. When to use versus ANOVA and ANCOVA
- B. Assumptions, assumptions
- C. Time getting involved
- D. Interpretation
- E. Post-hoc tests

Clustering and Trees

- A. When to use certain types
- B. Clarity and usefulness

Real World Examples

MANOVA

Abbas, M., Ebeling, A., Oelmann, Y., Ptacnik, R., Roscher, C., Weigelt, A., et al. (2013). Biodiversity Effects on Plant Stoichiometry. *PLoS One*, 8(3), e58179, doi:10.1371/journal.pone.0058179.

Since the six different molar ratios (C:N, C:P, C:K, N:P, N:K, P:K) were not independent of each other, a multivariate analysis of variance (MANOVA) was performed with the following factors: block, sown species richness, functional group richness, legume presence, grass presence.

	May 2003	May 2004	May 2005	May 2006	May 2007
Block	0.543* (CN,NP,CP,CK,NK)	0.638*** (CP,CK)	0.356. (CP,CK,NK)	0.415* (CP)	0.636*** (CN,CP,CK,NK,PK)
sown diversity	0.079	0.095	0.152. (PK)	0.226* (NP,CP,PK)	0.301*** (NP,CP,PK)
functional group richness	0.147	0.111	0.197* (CP,NK,PK)	0.167. (NP,CP)	0.296*** (CN,NP)
Legume	0.525*** (CN,NP,CK,NK,PK)	0.287*** (CN,NP,CK,NK,PK)	0.578*** (all)	0.696*** (all)	0.706*** (CN,NP,CK,NK,PK)
Grass	0.200. (CN)	0.320*** (CN,CP,PK)	0.223* (CN,CP,CK)	0.385*** (CN,CP,CK)	0.366*** (CN,CP,CK)

For each factor, the Pillai Trace value and its significance level are given as well as all ratios for which the factor effect was significant at $p < 0.05$. Significance levels: $p < 0.001 = ***$, $p < 0.01 = **$, $p < 0.05 = *$, $p < 0.1 = .$
 doi:10.1371/journal.pone.0058179.t001

Real World Examples

MANOVA

Roy, S., Lavine, J., Chiaromonte, F., Terwee, J., VandeWoude, S., Bjornstad, O., et al. (2009). Multivariate Statistical Analyses Demonstrate Unique Host Immune Responses to Single and Dual Lentiviral Infection. *PloS One*, 4(10), e7359, doi:10.1371/journal.pone.0007359.

To investigate significant effects of single and dual infection on the responses, we used Multivariate Analysis of Variance (MANOVA; [22], [23]) with tests based on Pillai's trace. Since the responses have a marked co-variation structure, these provide enhanced power relative to univariate tests assessing differences in infection group means separately for each response [24].

Symbol	Description
Th-1 and Th-2 cytokines:	Soluble factors modulating innate and adaptive immune response
IL-4	B-cell growth factor, 'Th2' cytokine
IL-10	B-cell survival and proliferation, 'Th2'. Generally antagonistic to TNF α
IL-12	Stimulates production of IFN- γ and TNF α , 'Th1'
TNF α	Stimulates systemic inflammation, regulates apoptosis, neutrophil chemoattractant
IFN γ	Proinflammatory cytokine, stimulates IL-12 and TNF α , antagonistic to IL-4, 'Th1'
FAS	'Death receptor', induces apoptosis
Circulating immunocytes:	Peripheral markers of immune homeostasis
Lymph	T and B Lymphocytes, NK cells and monocytes
CD4	Cell surface marker for T helper cells (lymph subset)
CD8	Cell surface marker for cytotoxic T cells (lymph subset)
CD25	Cell surface marker for activated T cells (both CD4 and CD8) and T regulatory cells
Neutr	Neutrophils; granular leukocytes, phagocytic. (innate immune system)

doi:10.1371/journal.pone.0007359.t001

	Day 31	Day 37	Day 52	Day 59
PLV	p = 0.006^a E = 0.95	P = 0.16 E = 0.81	p = 0.01 E = 0.93	p = 0.18 E = 0.80
FIVC	p = 0.52 E = 0.65	P = 0.07 E = 0.86	p = 0.02 E = 0.91	p = 0.04^b E = 0.89
PLV \times FIVC	p = 0.96 E = 0.36	P = 0.29 E = 0.75	p = 0.18 E = 0.80	p = 0.56 E = 0.63

^aSignificant results are in bold.
^bThis value is not significant if the MANOVA is run without imputing the values (p = 0.06).

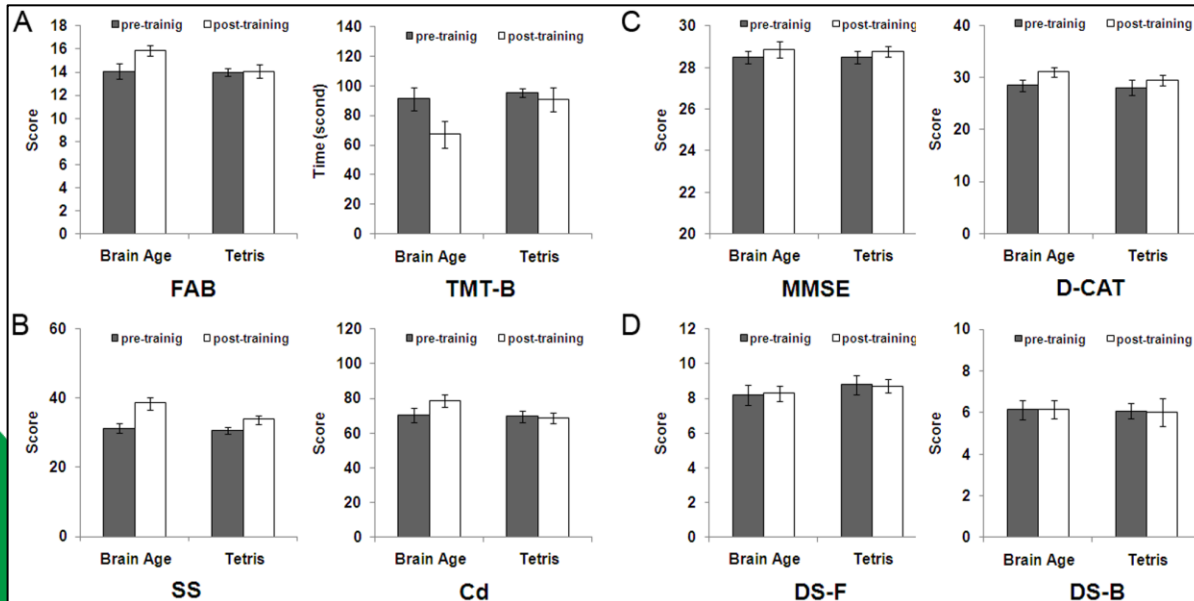
doi:10.1371/journal.pone.0007359.t004

Real World Examples

MANCOVA

Nouchi, R., Taki, Y., Takeuchi, H., Hashizume, H., Akitsuki, Y., Shigemune, Y., et al. (2012). Brain Training Game Improves Executive Functions and Processing Speed in the Elderly: A Randomized Controlled Trial. *PLoS One*, 7(1), e29676, doi:10.1371/journal.pone.0029676.

We conducted multivariate analyses of covariance (MANCOVA) for the change scores (post-training score minus pre-training score) in each of cognitive tests (Figure 2, Table 3). The change scores were the dependent variable, groups (Brain Age, Tetris) was the independent variable. Pre-training scores in all cognitive tests, sex, age, and education levels (years) were the covariate to exclude the possibility that any pre-existing difference of measure between groups affected the result of each measure and adjust for background characteristics.



	Brain Age Group		Tetris Group		Effect size (η^2)	p-value
	Mean	SD	Mean	SD		
Executive function						
FAB (score)	1.79	(1.58)	0.07	(1.21)	0.13	0.001
TMT-B (seconds)	-24.00	(22.81)	-4.57	(22.32)	0.13	0.006
Attention						
D-CAT (number)	2.57	(4.36)	1.43	(3.11)	0.06	0.277
DS-F (low score)	0.07	(1.94)	-0.07	(1.86)	0.00	0.717
DS-B (low score)	0.00	(1.41)	-0.07	(1.90)	0.00	0.683
Global cognitive status						
MMSE (score)	0.36	(1.28)	0.29	(1.33)	0.00	0.631
Processing speed						
Cd (number)	8.29	(7.03)	-0.93	(8.08)	0.19	0.005
SS (number)	7.43	(4.91)	3.21	(5.13)	0.12	0.014

Change scores were calculated by subtracting the pre-cognitive measure score from the post-cognitive measure score. The Executive functions were measured by frontal assessment battery at bedside (FAB) and trail making test type B (TMT-B). The processing speeds were measured by digit symbol coding (Cd) and symbol search (SS). The global cognitive status was measured by mini-mental state examination (MMSE). The attention was measured by digit cancellation task (D-CAT), digit span forward (DS-F) and digit span backward (DS-B). We report eta square (η^2) as an index of effect size. It is a standardized difference in the change score (post-training score minus pre-training score) between intervention groups (Brain Age, Tetris). $\eta^2 \geq .01$ is regarded as small effect, $\eta^2 \geq .06$ as medium effect, and $\eta^2 \geq .14$ as large effect. SD means standard deviation.

doi:10.1371/journal.pone.0029676.t003

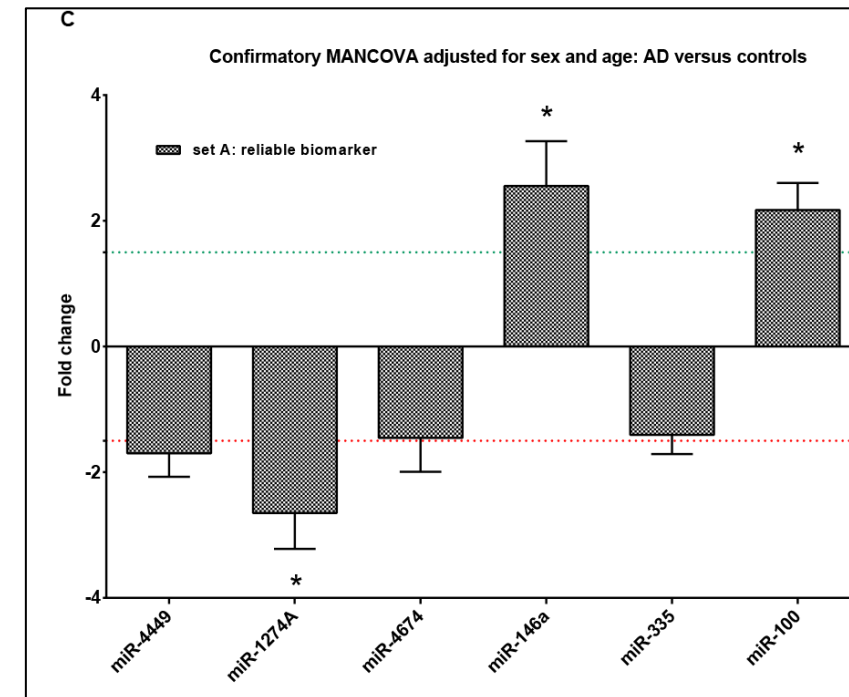
Real World Examples



MANCOVA

Denk, J., Boelmans, K., Siegismund, C., Lassner, D., Arlt, S., & Jahn, H. (2015). MicroRNA Profiling of CSF Reveals Potential Biomarkers to Detect Alzheimer`s Disease. *PloS One*, 10(5), e0126423, doi:10.1371/journal.pone.0126423.

After identifying the reliable biomarker candidates of set A and the most informative variables of set B, inferential statistics followed by applying multivariate analyses of covariance (MANCOVA) with sex and age as covariates. Those miRNAs among the biomarker candidates, which revealed significant differences between the AD and control group after Bonferroni adjustments on the confirmatory level, were designated as significant biomarkers.



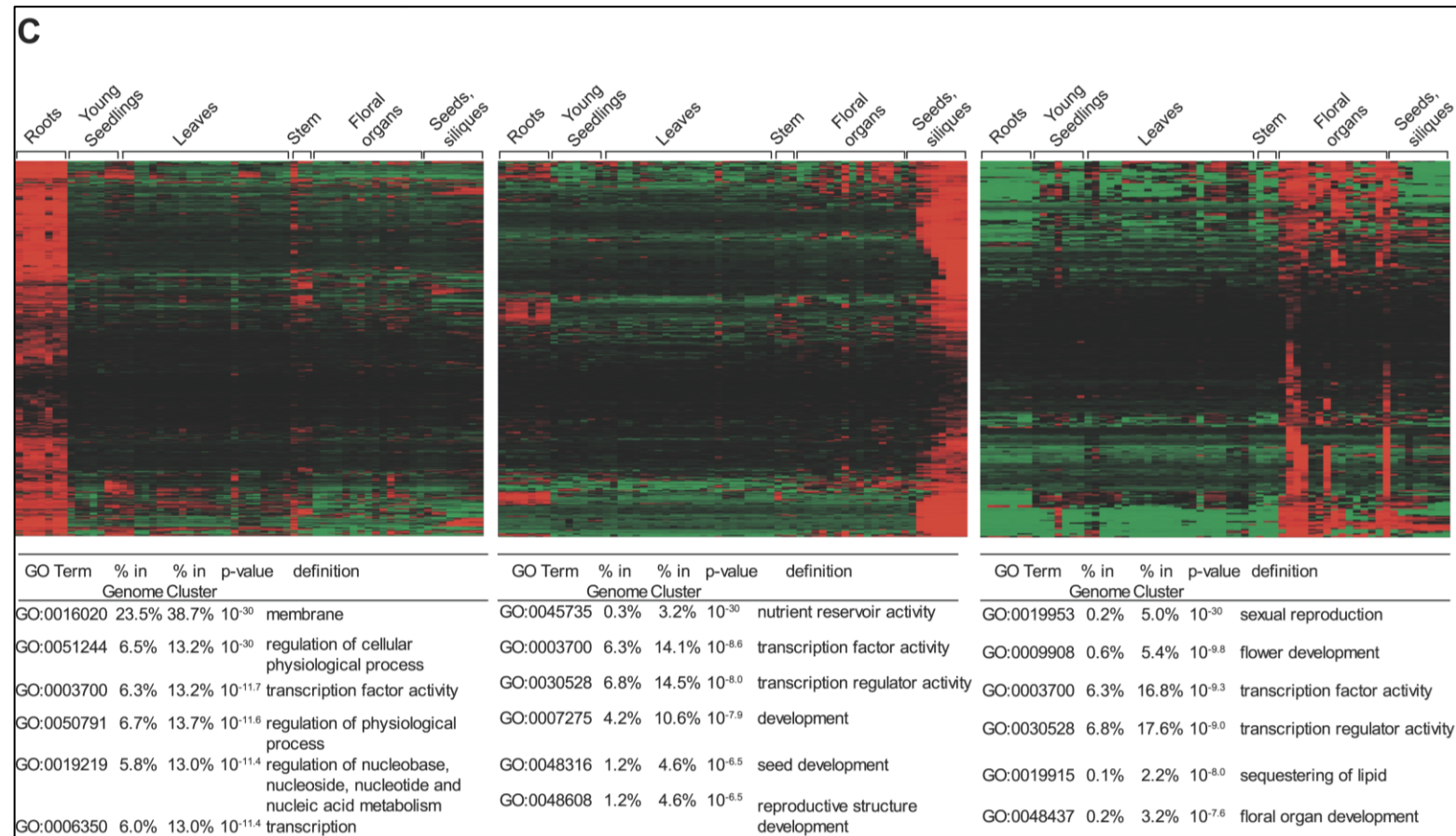
(C) Bar diagram of the reliable biomarker signals of set A. Stars (*) over the bars point to significant p-values (MANCOVA, $p < \alpha^*$, where α^* is Bonferroni corrected $\alpha = 0.05$) and therewith to significant biomarkers.

Real World Examples

K-means clustering

Zhang, X., Clarenz, O., Cokus, S., Bernatavichute, Y. V., Pellegrini, M., Goodrich, J., et al. (2007). Whole-Genome Analysis of Histone H3 Lysine 27 Trimethylation in Arabidopsis. PLoS Biology, 5(5), e129, doi:10.1371/journal.pbio.0050129.

For cluster analysis, the logarithm of the expression ratio for each gene divided by its mean value across all conditions was computed. This data was then clustered into 8–10 mutually exclusive groups using K-means clustering [50]. The genes within each cluster were then hierarchically clustered and displayed in the figures.



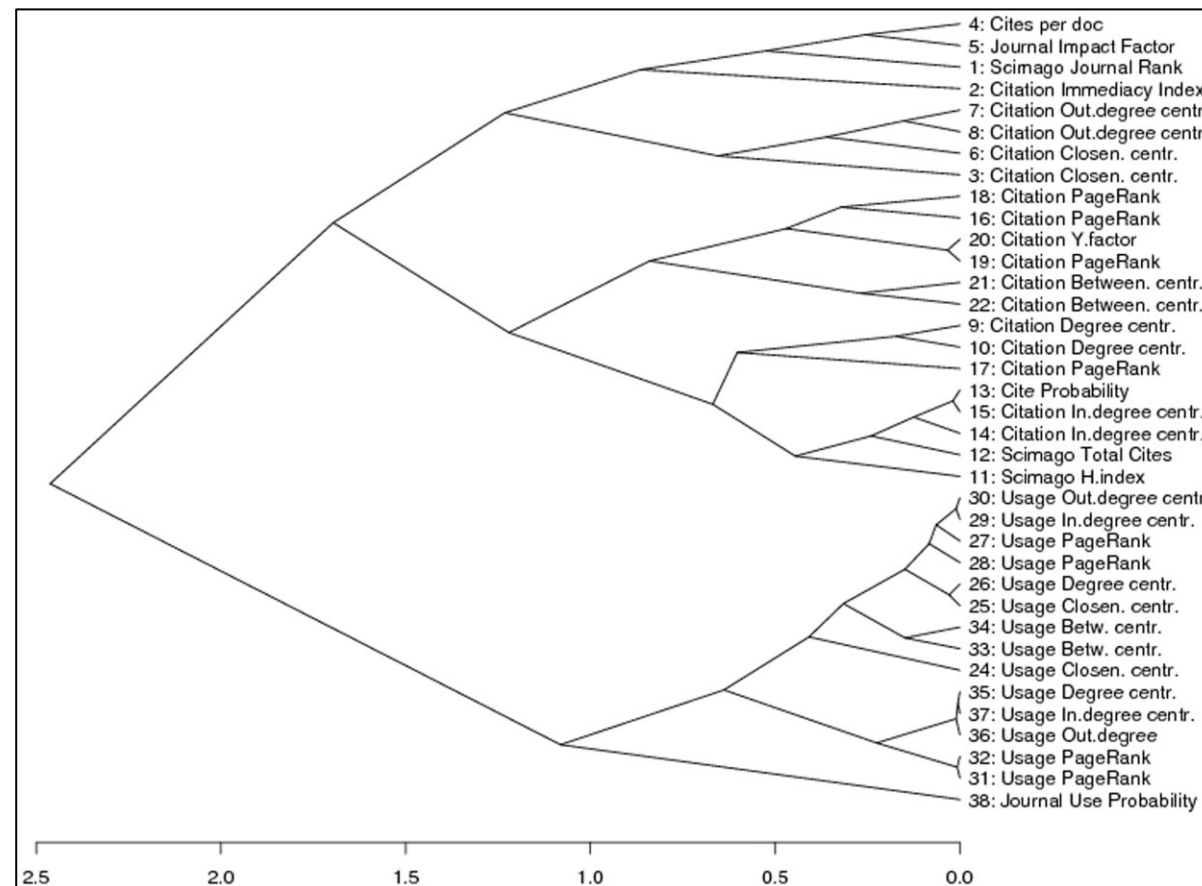
Real World Examples

Hierarchical clustering

Bollen, J., Van de Sompel, H., Hagberg, A., & Chute, R. (2009). A Principal Component Analysis of 39 Scientific Impact Measures. PloS One, 4(6), e6022, doi:10.1371/journal.pone.0006022.

To cross-validate the PCA results, a hierarchical cluster analysis (single linkage, euclidean distances over row vectors) and a k-means cluster analysis were applied to the measure correlations in to identify clusters of measures that produce similar journal rankings.

Cluster	Measures	Interpretation
1	38	Journal Use Probability
2	24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37	Usage measures
3	1, 2, 3, 4, 5	JIF, SJR, Cites per Document measures
4	6, 7, 8, 9, 10, 11, 12, 13, 14, 15	Total Citation rates and distributions
5	16, 17, 18, 19, 20, 21, 22	Citation Betweenness and PageRank



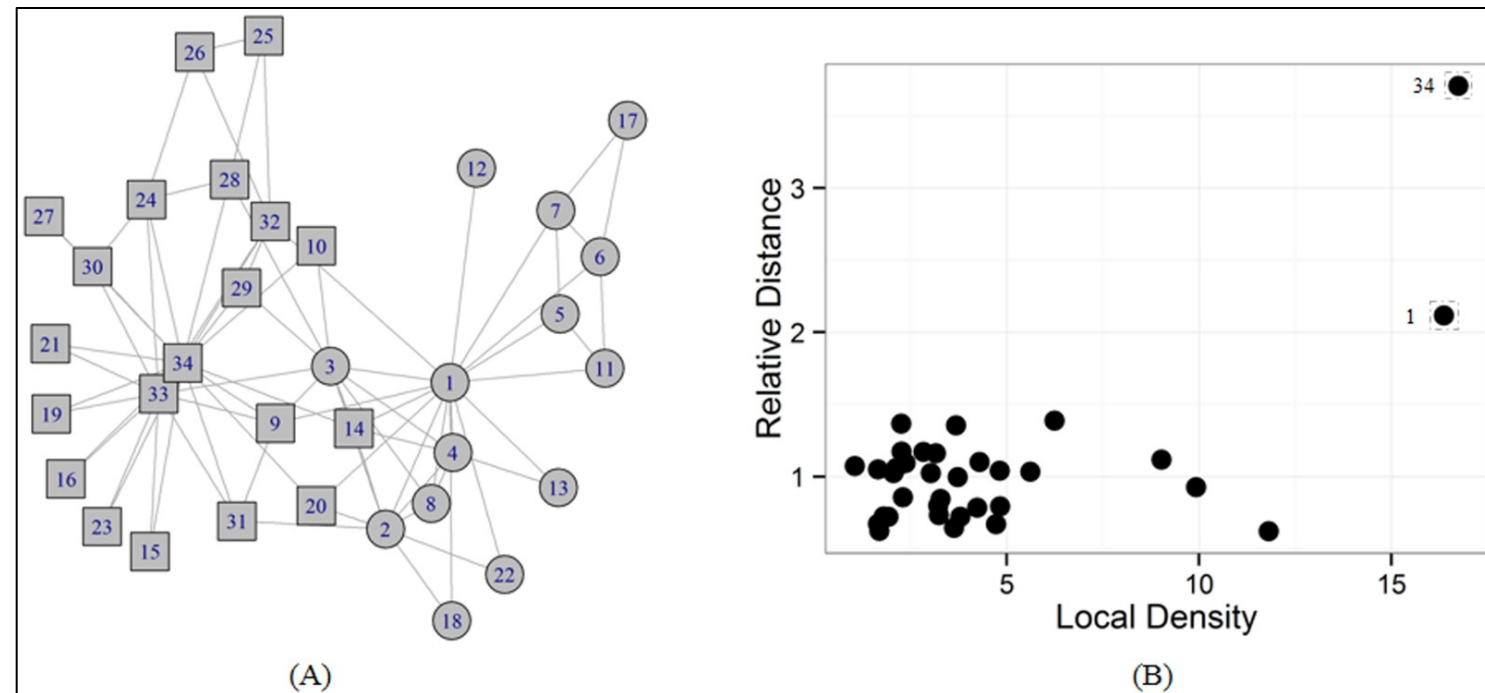
Real World Examples

Density-based clustering

Wang X, Liu G, Li J, Nees JP (2017) Locating Structural Centers: A Density-Based Clustering Method for Community Detection. PLoS ONE 12(1): e0169355.
<https://doi.org/10.1371/journal.pone.0169355>

In this work, we present a new method for community detection which is termed as LCCD. It is a density-based clustering method, inspired by recent research on data analysis [32] where data points are clustered by finding the cluster centers.

This observation is illustrated in Fig 1 by the Zachary's karate club network [42] that is a real-world social network. This interactive network with 34 nodes, ultimately split into two distinct groups, because of a disagreement between the administrator (vertex 1) and the instructor (vertex 34), as shown in Fig 1(A).

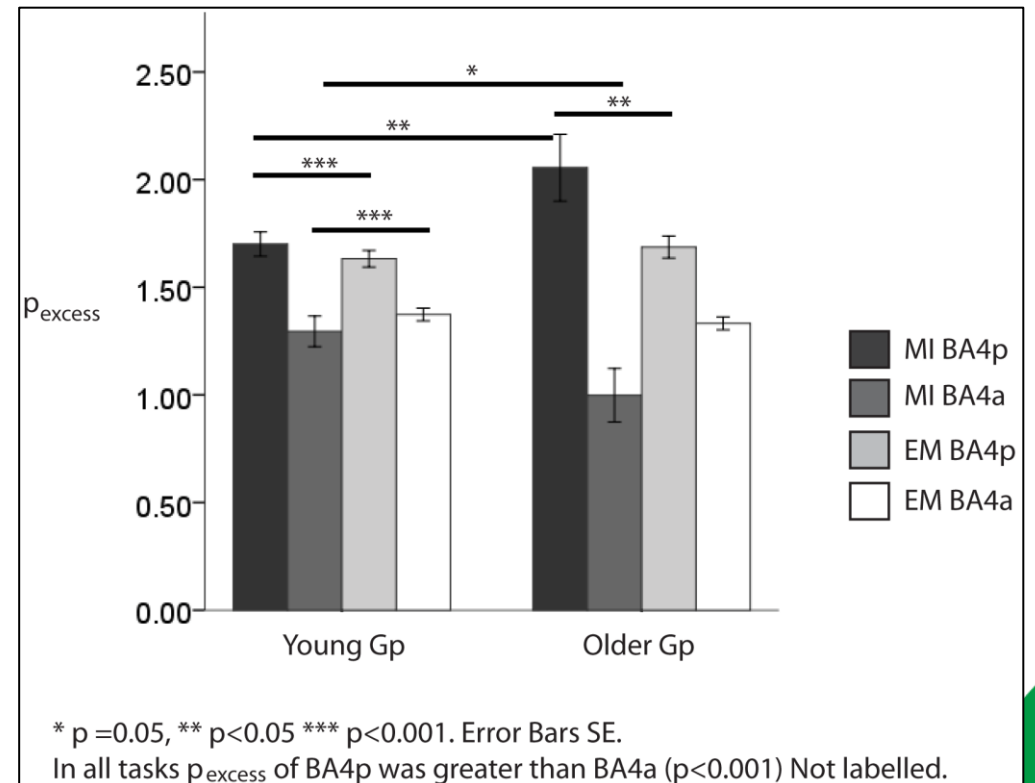


Real World Examples

Distribution-based clustering

Sharma N, Baron J-C (2014) Effects of Healthy Ageing on Activation Pattern within the Primary Motor Cortex during Movement and Motor Imagery: An fMRI Study. PLoS ONE 9(6): e88443. <https://doi.org/10.1371/journal.pone.0088443>

We explored the distribution-based clustering and weighted laterality index within BA4a and BA4p. The involvement of BA4p during MI (measured with distribution-based clustering) was significantly greater in the older group ($p < 0.05$) than in the younger group.

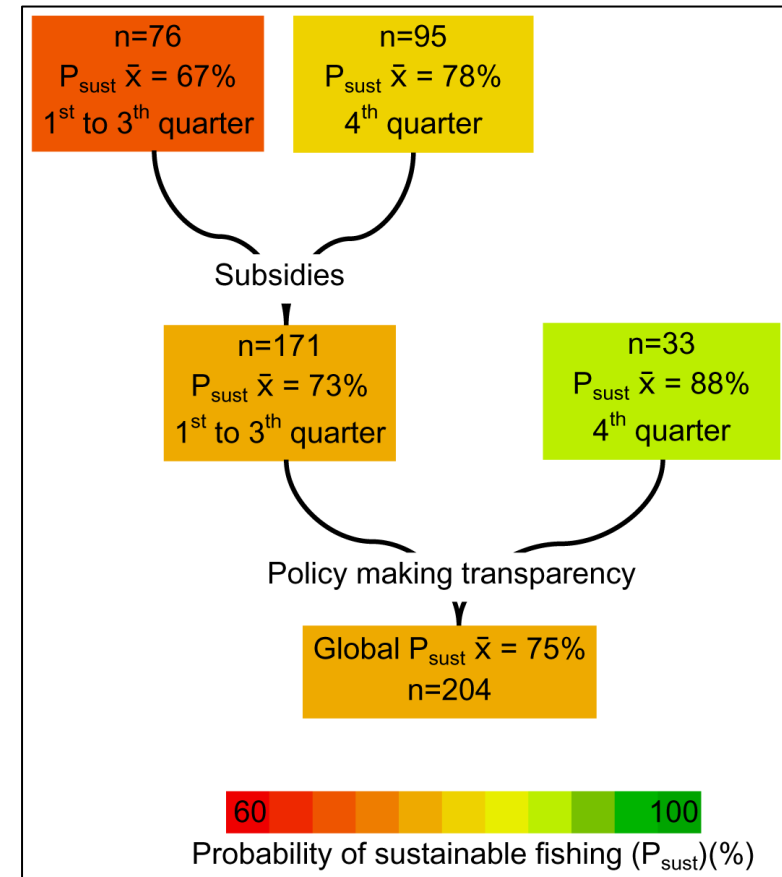


Real World Examples

Classification Tree

Mora, C., Myers, R. A., Coll, M., Libralato, S., Pitcher, T. J., Sumaila, R. U., et al. (2009). Management Effectiveness of the World's Marine Fisheries. *PLoS Biology*, 7(6), e1000131, doi:10.1371/journal.pbio.1000131.

Data on fisheries sustainability was quantified for the year 2004 and linked to the effectiveness of fisheries management using a classification/regression tree. A classification tree tests for significant differences in fisheries sustainability among the quarters of each attribute (note that the first and fourth quarters are the extremes of a scale from worst- to best-case scenarios for each attribute)



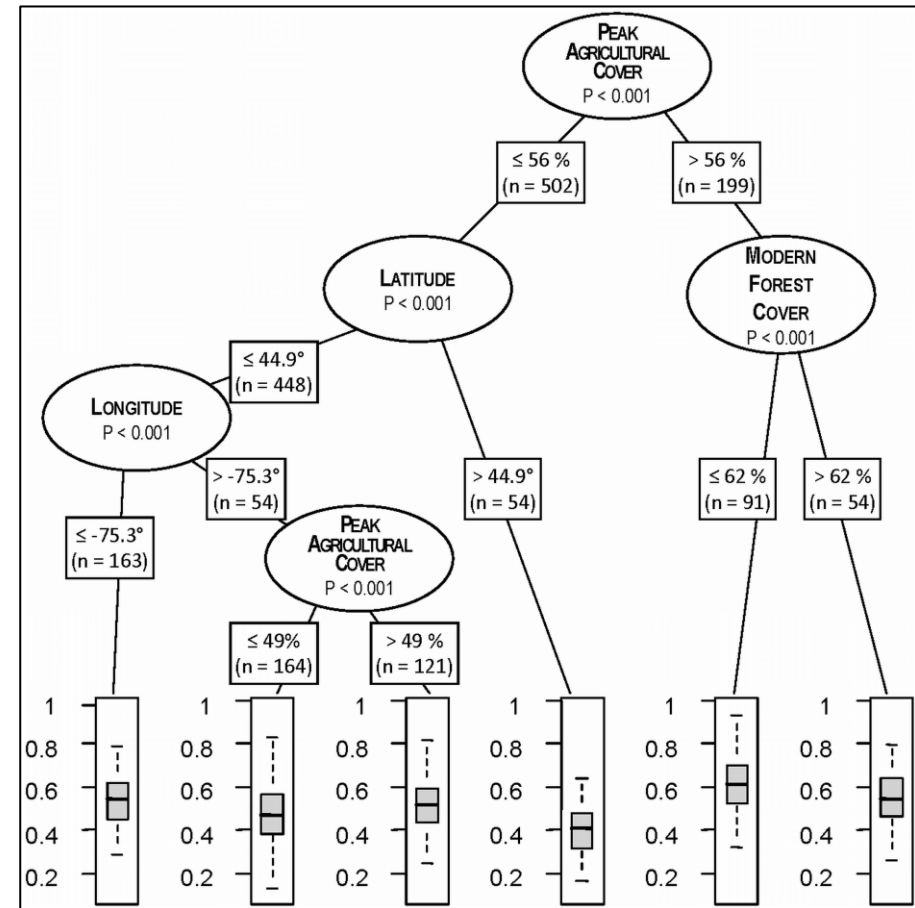
Real World Examples



Regression Tree

Thompson, J. R., Carpenter, D. N., Cogbill, C. V., & Foster, D. R. (2013). Four Centuries of Change in Northeastern United States Forests. *PloS One*, 8(9), e72540, doi:10.1371/journal.pone.0072540.

Finally, to evaluate the relationships between compositional change and the suite of predictor variables identified in Table 2 further, we used regression tree analysis (RTA) with the Sørensen's distance between time periods as the response variable.



Summary and Conclusion



- MANOVA and MANCOVA are extension of ANOVA and ANCOVA
 - Both involve several assumptions that need to be tested
 - The actual analysis is straightforward
 - Results are usually in table form
- Clustering comes in a variety of methods
 - Used to classify observations into responses based on information
 - Often can be graphed
- Tune in next time for a plunge into advanced topics of Multivariate Analysis Module III: Deep Dive

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