Multilevel Modeling for the Uninitiated

BERDC Special Topics Talk 13

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DAKOTA COMMUNITY COLLABORATIVE ON TRANSLATIONAL ACTIVITY

Dr. Mark Williamson

Biostatistics, Epidemiology, and Research Design Core

Opening

Goal: Gaze at the expansive rafters of multilevel modeling

- Definitions and common types — ***'A View from Above'***
- Designing and setting up models — ***'Blueprints and Bricks'***
- Results and Caveats — ***'Opening the Doors Carefully'***
- Conceptual Examples — ***'An Awe in-Spiring Tour'***
- Worked Examples — ***'Going the Distance'***

Before Moving On:

Pre-test: https://und.qualtrics.com/jfe/form/SV_4VDpoMi807fkiTs

R code: https://med.und.edu/daccota/_files/docs/berdc_docs/multilevel_modeling_rcode.txt

SAS code: https://med.und.edu/daccota/_files/docs/berdc_docs/multilevel_modeling_sascode.txt

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Definitions

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- "A multilevel model is a statistical model applied to data collected at more than one level in order to elucidate relationships at more than one level" [1-2]
- Put simply, multiple levels of data
- Also known as: Linear Mixed Models, Mixed Effects Models, Hierarchical Models, Nested Models, Repeated Measures Models, Random Effects Models, Random Coefficient Models… [3]
- Always have both fixed and random effects

Types

Two broad **approaches**:

- Multiple regression
- Structural Equation Modeling (SEM)

Three basic **classes**:

- Unconstrained (null)
- Random Intercepts
- Random Intercepts and Slopes

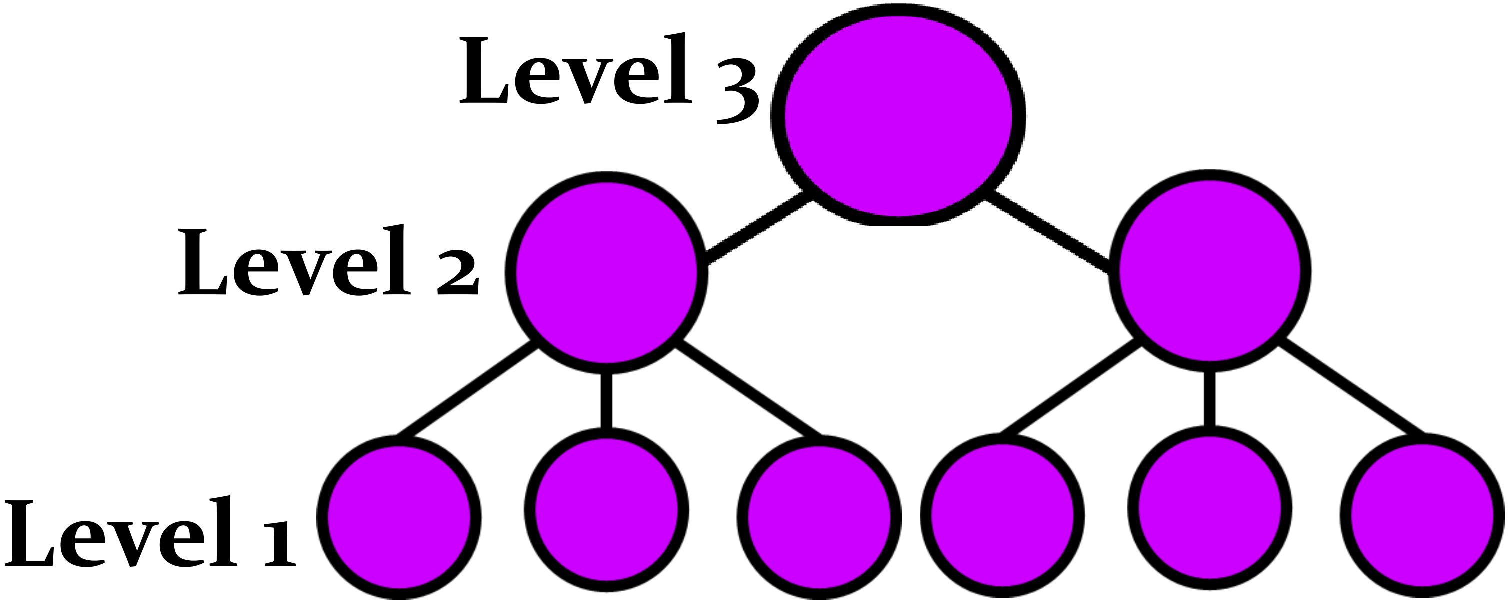
Common **Extensions**:

- 3-Level Models
- Non-normal
- Longitudinal
- Meta-analysis

Levels

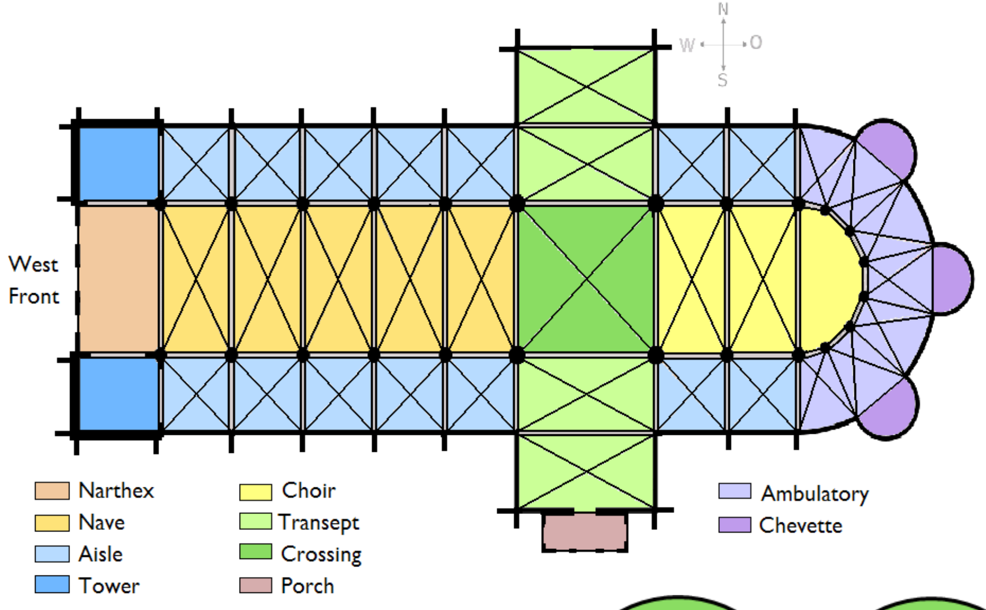
Examples of Levels:

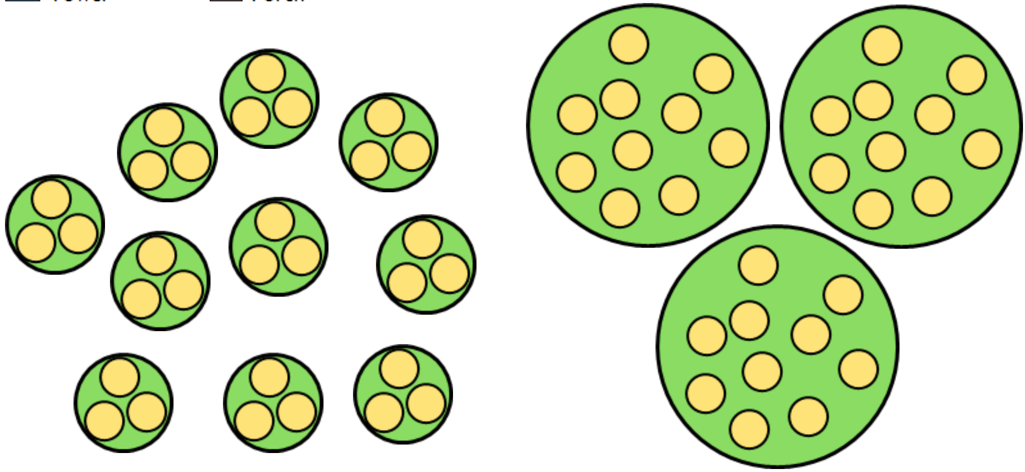
- Basic 2-Level: Individuals nested in Classes
- 3-Level: Individuals nested in Classes nested in Schools
- Longitudinal: Repeated measurements nested in Individuals



Design 1

- Study Design Blueprint[4]
  - Sample representation (L1, L2, …)
  - Number of groups (L2 cluster size)
  - Cohort vs. Cross-sectional
  - Randomization (experimental)
- Data Design
  - Levels
  - Variables



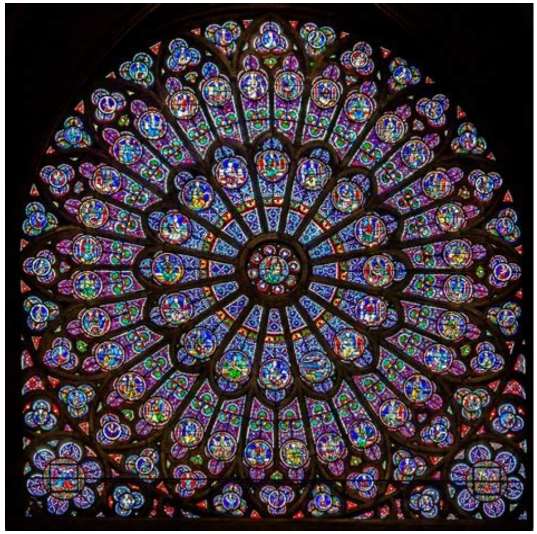
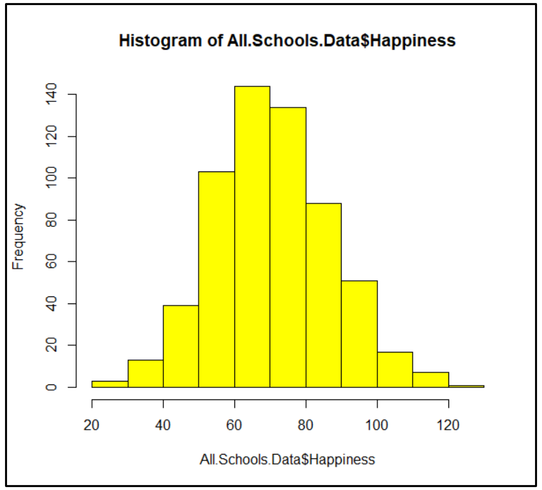


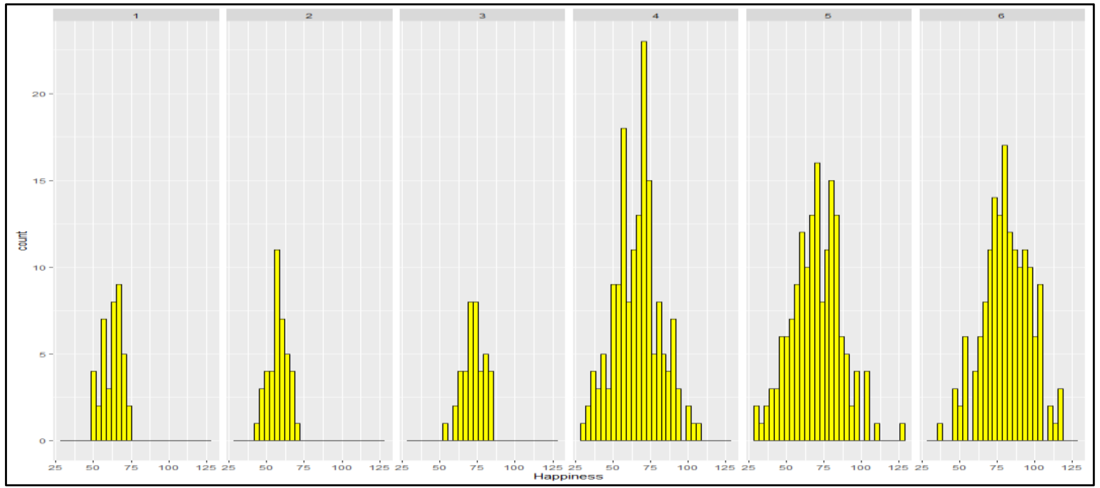
Design 2

- Visualizing Data [5]
  - Samples and Summaries
  - Histograms
  - Correlation
  - Scatter plots
  - Spaghetti plots

##	Happiness	Friends	GPA	StudentID
## 1	54.60136	12.741917	2.744812	1
## 2	64.21655	8.870604	1.473716	2
## 3	62.91056	10.726257	2.077085	3
## 4	66.52306	11.265725	2.936896	4
## 5	57.07570	10.808537	3.327470	5
## 6	68.82087	9.787751	2.690941	6

Happiness	Friends	GPA	StudentID	School
Min. : 29.36	Min. :-0.1076	Min. :1.001	Min. : 1.0	Min. :1.0
1st Qu.: 59.21	1st Qu.: 4.0221	1st Qu.:1.633	1st Qu.:150.8	1st Qu.:4.0
Median : 69.88	Median : 5.4308	Median :2.458	Median :300.5	Median :5.0
Mean : 70.68	Mean : 5.9057	Mean :2.454	Mean :300.5	Mean :4.4
3rd Qu.: 81.38	3rd Qu.: 7.5602	3rd Qu.:3.267	3rd Qu.:450.2	3rd Qu.:6.0
Max. :126.32	Max. :15.4038	Max. :3.998	Max. :600.0	Max. :6.0



Design 2

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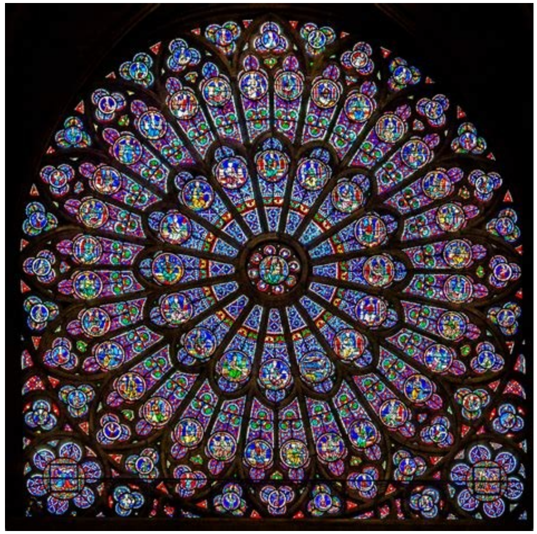

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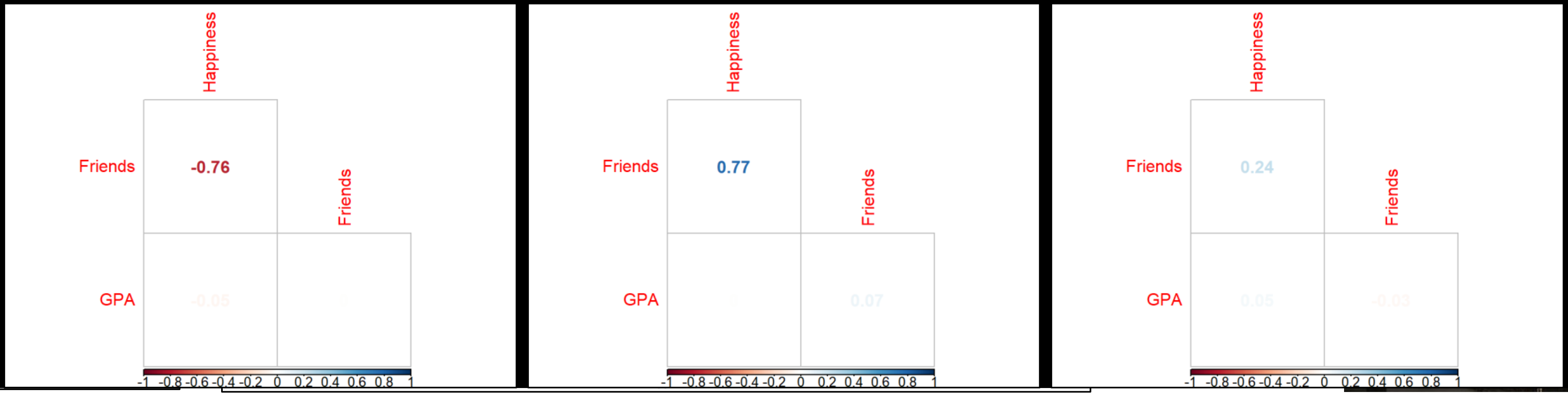

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Design 2

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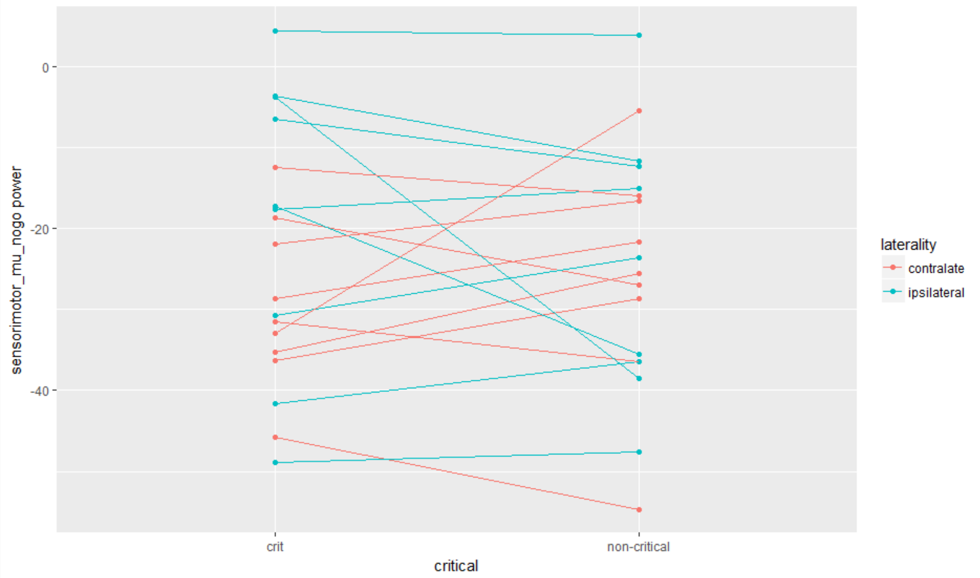
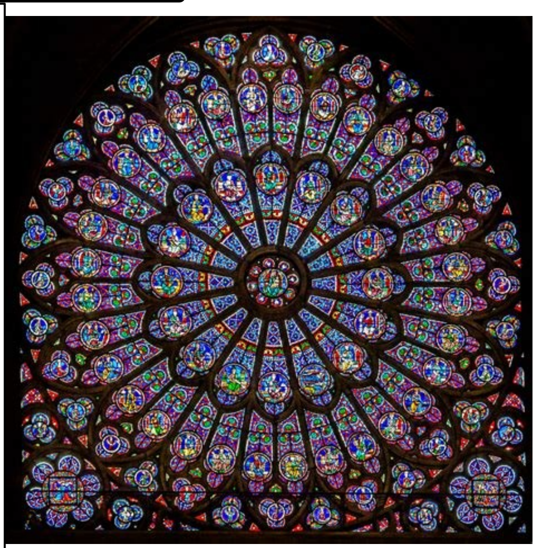
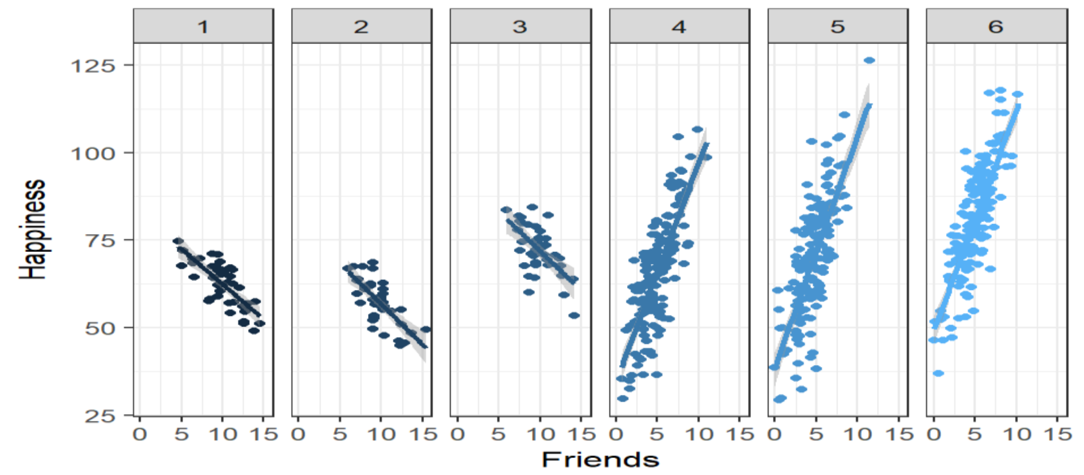
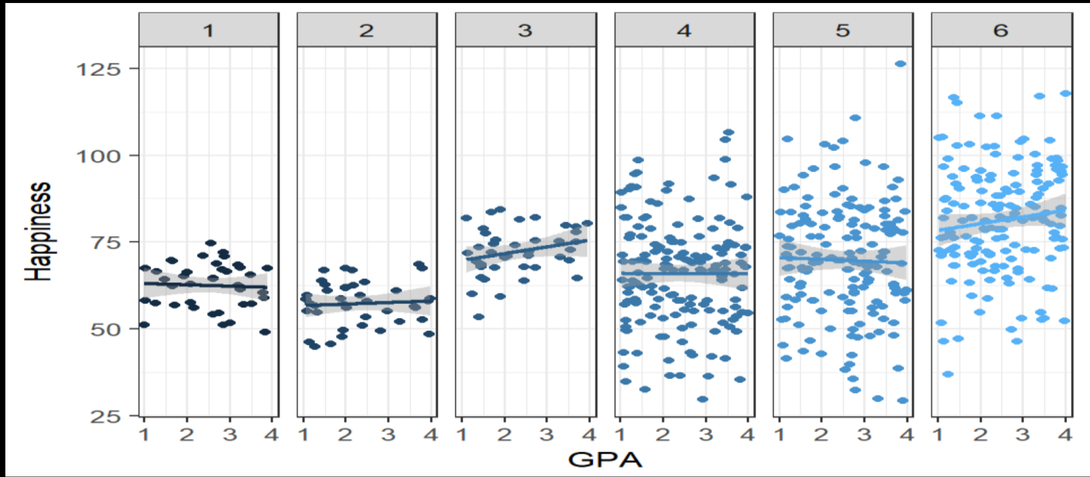
- Visualizing Data [5]
  - Samples and Summaries
  - Histograms
  - Correlation
  - Scatter plots
  - Spaghetti plots

```
  Happiness
Min.   : 29.36    Mi
1st Qu.: 59.21    1s
Median : 69.88    Me
Mean   : 70.68    Me
3rd Qu.: 81.38    3r
Max.   :126.32    Ma
```





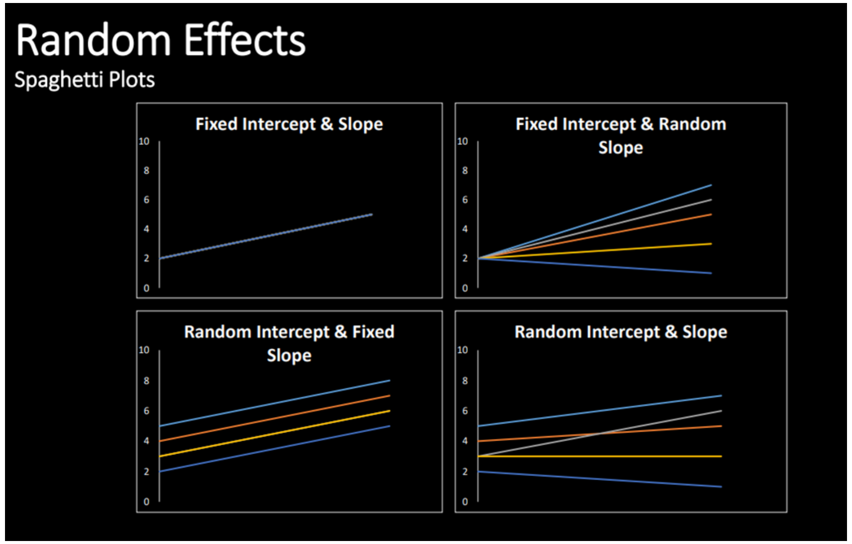


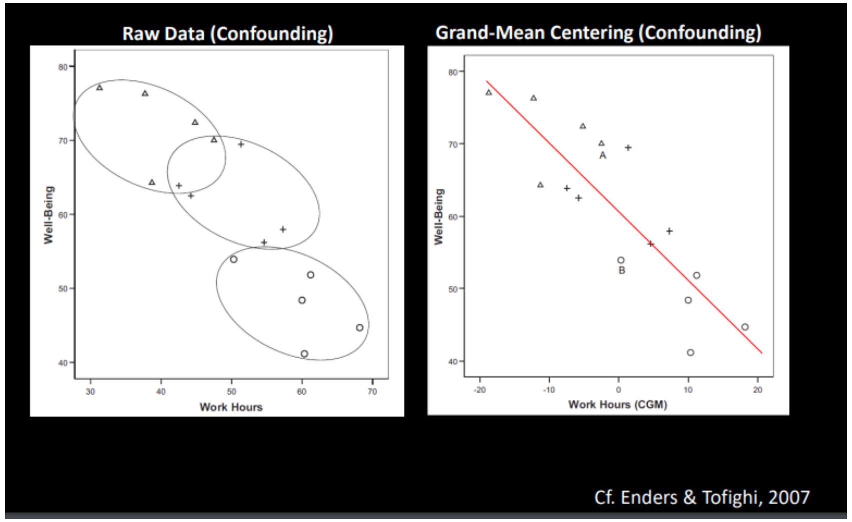


Setup 1

- Crafting Model [6-7]
  - From bottom up (no 'best' approach)
  - Formulate equation (if that helps you)
  - Fixed/Random Intercepts & Slopes
  - Centering
  - Covariance matrices
  - Maximum Likelihood (ML) versus Restricted Maximum Likelihood (REML)

$$Y_{ij} = \beta_{0j} + \beta_{1j}X_{ij} + e_{ij}$$
$$\beta_{0j} = \gamma_{00} + \gamma_{01}W_j + u_{0j}$$
$$\beta_{1j} = \gamma_{10} + u_{1j}$$





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Setup 2

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- Getting your computer to do your whims [7]

Sample Syntax: R

2-LEVEL

- Lmer

model <-lmer(*DV* ~ *Predictor1* + (1 + *Predictor1*|*ID*), data=*data*, na.action = "na.exclude")

summary (model)

- Nlme

model <- lme(*DV* ~ *Predictor1*, random = ~*Predictor1*|*ID*, na.action = "na.exclude", data=data)

summary(model)

Sample Syntax: R

3-LEVEL

- Lmer

model <-lmer(*DV* ~ *Predictor1* + (1 + *Predictor1*|*ID:DAY*) + (1 + *Predictor1*|*ID*), data=data, na.action = "na.exclude")

summary (model)

- Nlme

model <- lme(*DV* ~ *Predictor1*, random = list(*ID* = ~*Predictor1*, *DAY* = ~*Predictor1*), na.action = "na.exclude", data=data)

summary(model)

Sample Syntax: R

CROSS-CLASSIFIED

- Lmer

model <-lmer(*DV* ~ *Predictor1* + (1 + *Predictor1*|*ID*) + (1 + *Predictor1*|*Target*), data=*data*, na.action = "na.exclude")

summary (model)

- Nlme

Crossed models are slow

Sample Syntax: SAS

2-Level

PROC MIXED data=*data* COVTEST;

CLASS *ID*;

MODEL *DV* = *Predictor1*/CL S DDFM=satterth;

RANDOM INTERCEPT *Predictor1* / SUB=*ID* TYPE=UN;

RUN;

Sample Syntax: SAS

3-Level

PROC MIXED data=*data* COVTEST;

CLASS *DAY ID*;

MODEL *DV* = *Predictor1*/CL S DDFM=satterth;

RANDOM INTERCEPT *Predictor1* / SUB=*DAY(ID)* TYPE=UN;

RANDOM INTERCEPT *Predictor1* / SUB=*ID* TYPE=UN;

RUN;

Sample Syntax: SAS

CROSS-CLASSIFIED

PROC MIXED data=*data* COVTEST;

CLASS *ID TARGET*;

MODEL *DV* = *Predictor1*/CL S DDFM=satterth;

RANDOM INTERCEPT *Predictor1* / SUB=*ID* TYPE=UN;

RANDOM INTERCEPT *Predictor1* / SUB=*TARGET* TYPE=UN;

RUN;

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Analyzing Results

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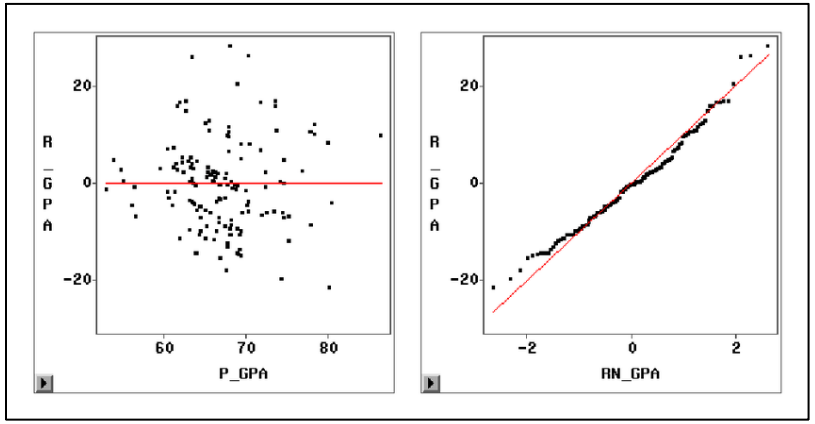
- Fixed Effects
  - Coefficients
  - Significance
- Random Effects
- Overall Model
  - AIC or BIC
  - Chi-Sq. /DF

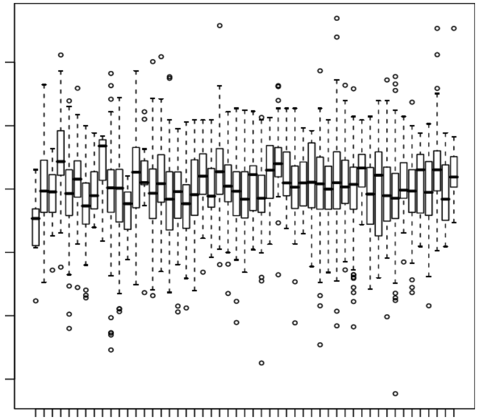
```
## Linear mixed model fit by maximum likelihood  ['lmerMod']
## Formula: Happiness ~ Friends.C + GPA.C + (1 + Friends.C | School)
##    Data: All.Schools.Data
##
##      AIC      BIC   logLik deviance df.resid
##   4438.4   4469.2  -2212.2   4424.4      593
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.6347 -0.6060  0.0229  0.6675  3.7765
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  School   (Intercept) 42.14    6.492
##           Friends.C   17.94    4.235    0.51
##  Residual             86.37    9.294
## Number of obs: 600, groups:  School, 6
##
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept)  75.3044     2.7888  27.003
## Friends.C     2.1566     1.7431   1.237
## GPA.C         0.4235     0.4271   0.991
##
## Correlation of Fixed Effects:
##           (Intr) Frnd.C
## Friends.C 0.454
## GPA.C     0.002  0.001
```

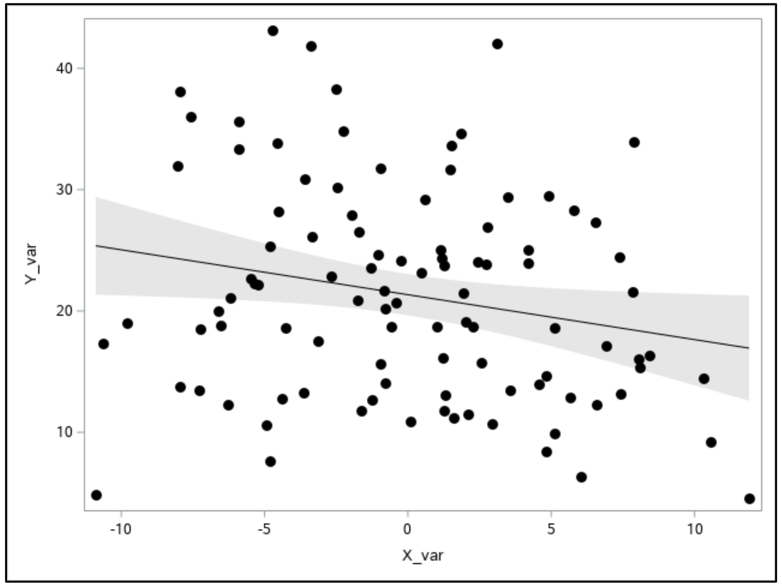
```
The GLIMMIX Procedure

     Convergence criterion (PCONV=1.11022E-8) satisfied.

          Fit Statistics

-2 Log Pseudo-Likelihood      40985.43
Generalized Chi-Square         6980.42
Gener. Chi-Square / DF            0.81

      Covariance Parameter Estimates

                                          Standard
Cov Parm      Subject       Estimate         Error

Intercept     SCHOOLID        1.1400        0.1094

             Solutions for Fixed Effects

                          Standard
Effect        Estimate       Error      DF    t Value    Pr > |t|

Intercept      -1.9594     0.07137     410     -27.45      <.0001
SEX             0.4668     0.06306    8170       7.40      <.0001

         Type III Tests of Fixed Effects

               Num      Den
Effect          DF       DF    F Value    Pr > F

SEX              1     8170      54.80    <.0001
```

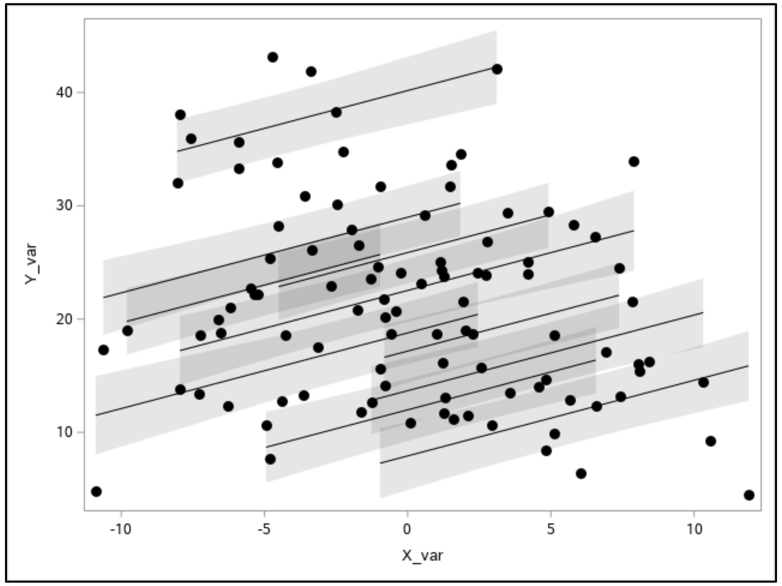
Analyzing Results 2

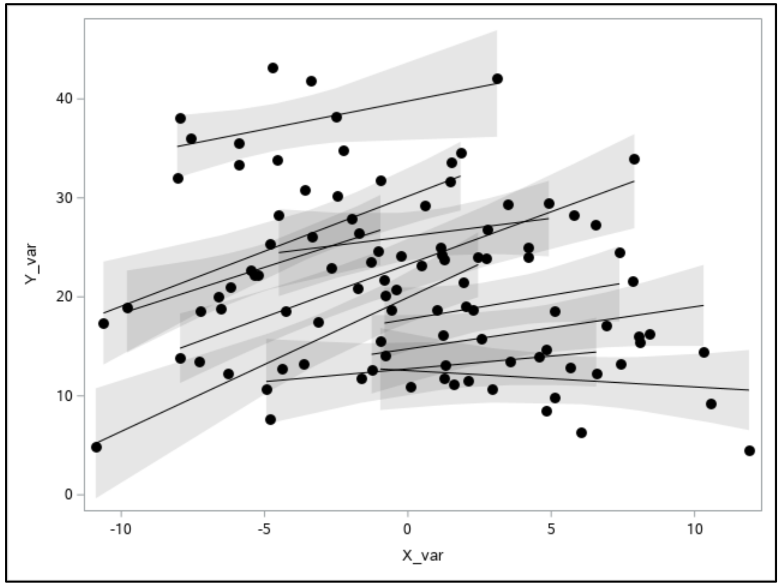
- Residuals
- QQ-plots
- Scatter plots with regression lines











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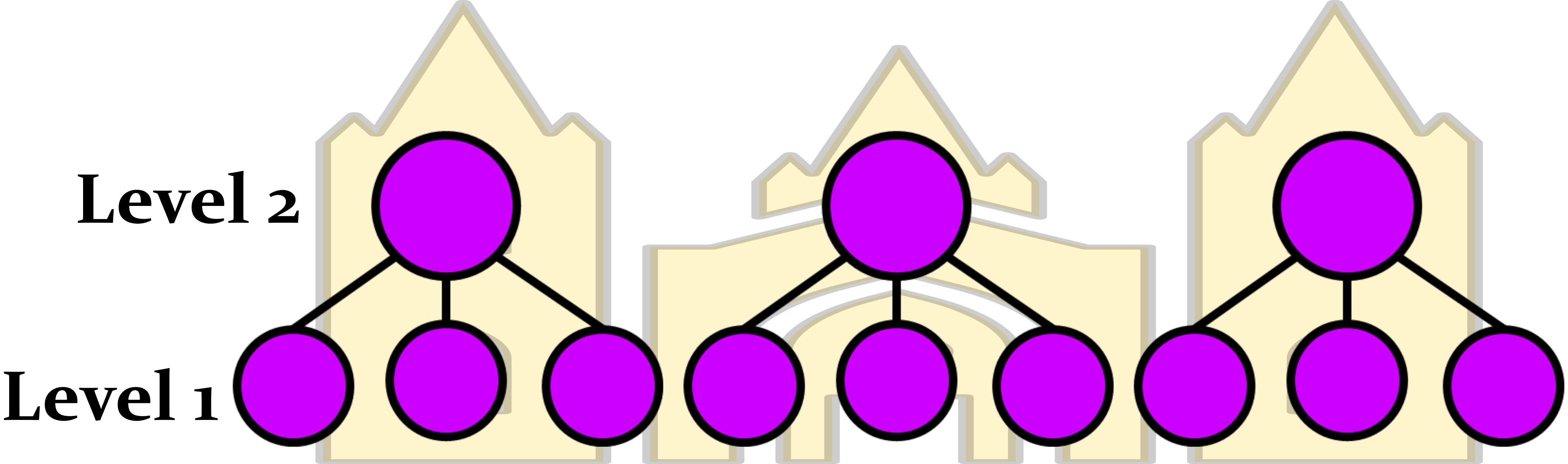
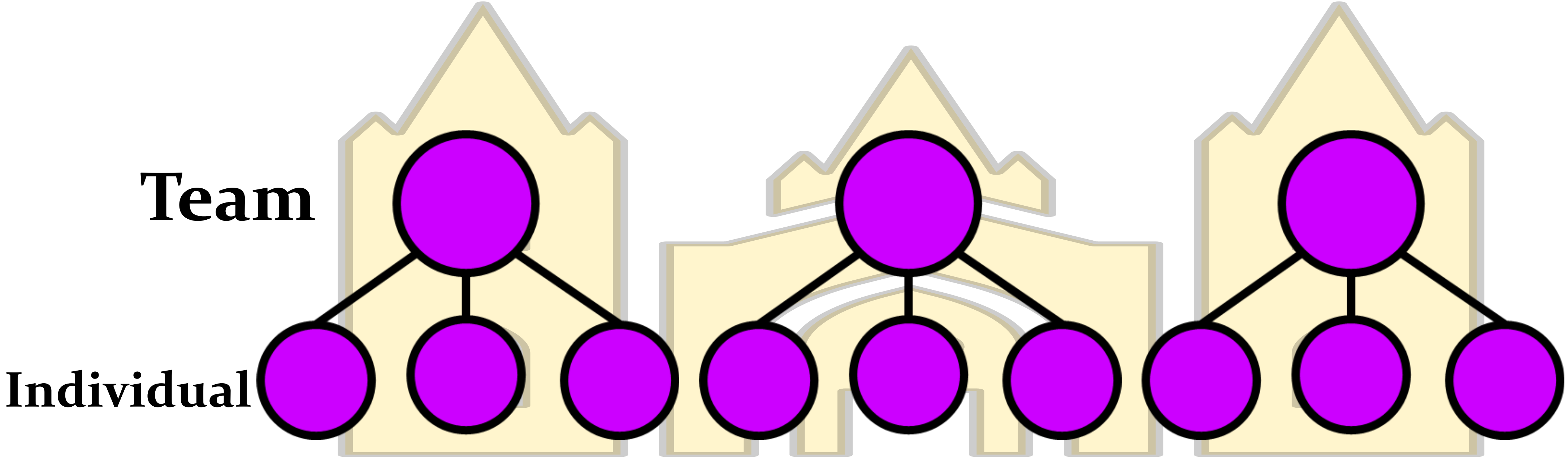
Caveats & Concerns

- When and when not [8]
  - **Yes**: correlated groupings (clusters), longitudinal
  - **No**: ICC too low, fixed effects approach good enough
  - Empirical, statistical, and theoretical justifications
- Reminders
  - Can treat intercepts and slopes as outcomes of level-2 predictors
  - Centering, covariance structures, and other options should be examined
  - Beware of non-normality

Conceptual Examples



Conceptual Examples



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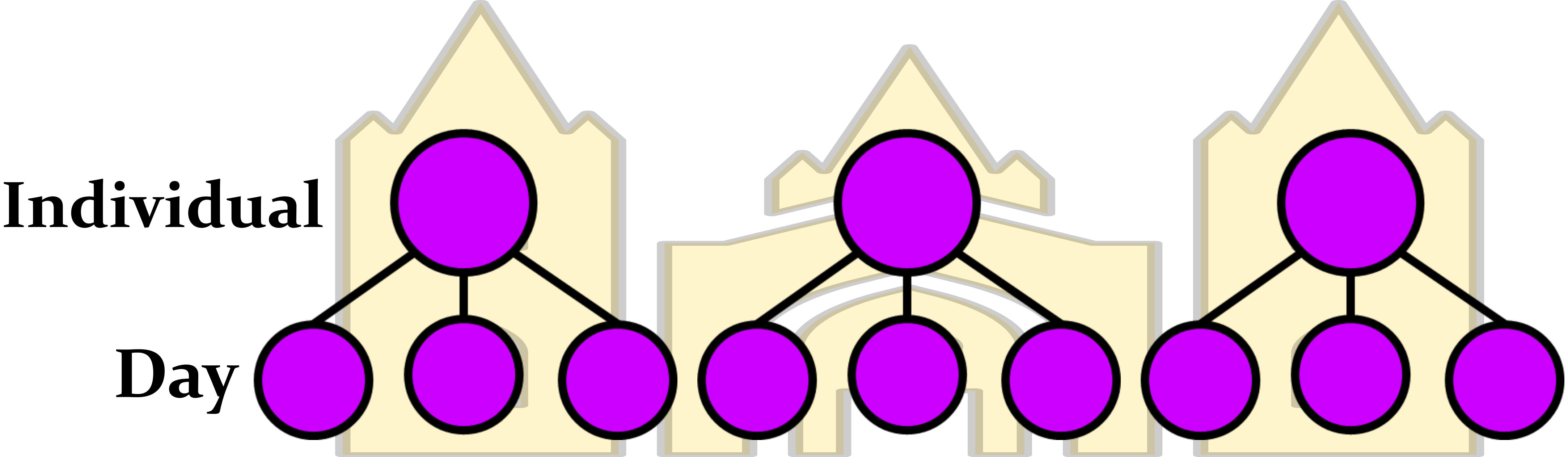

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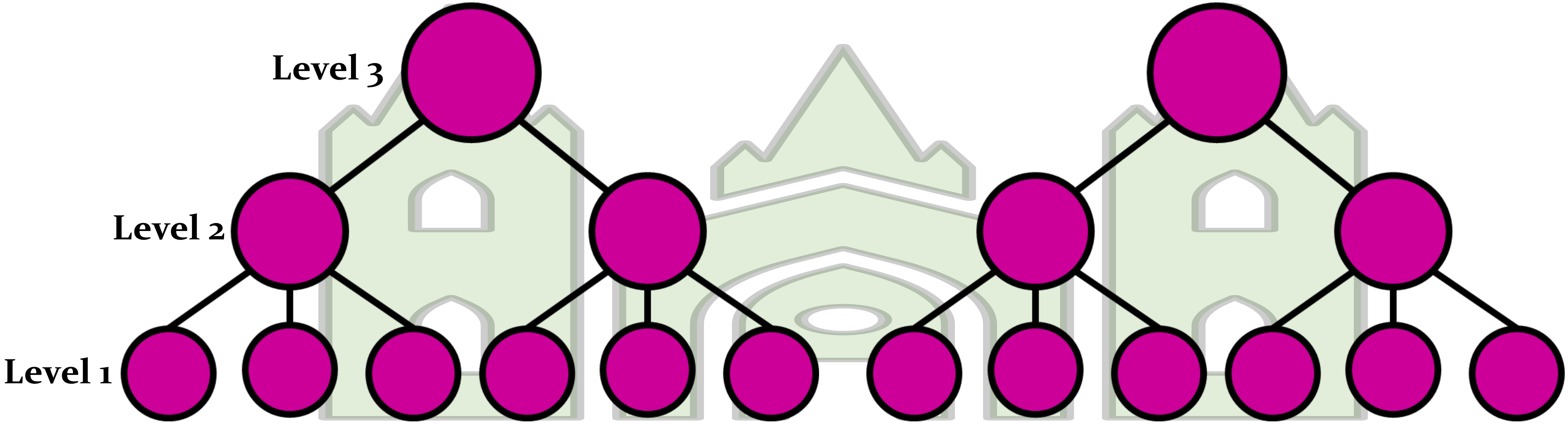

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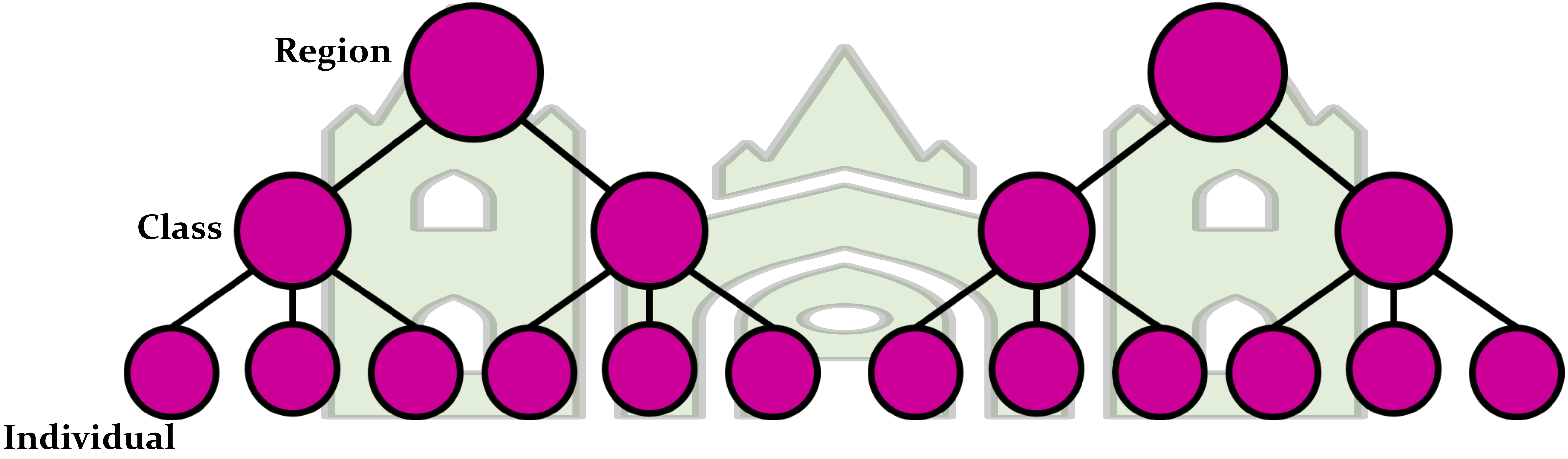

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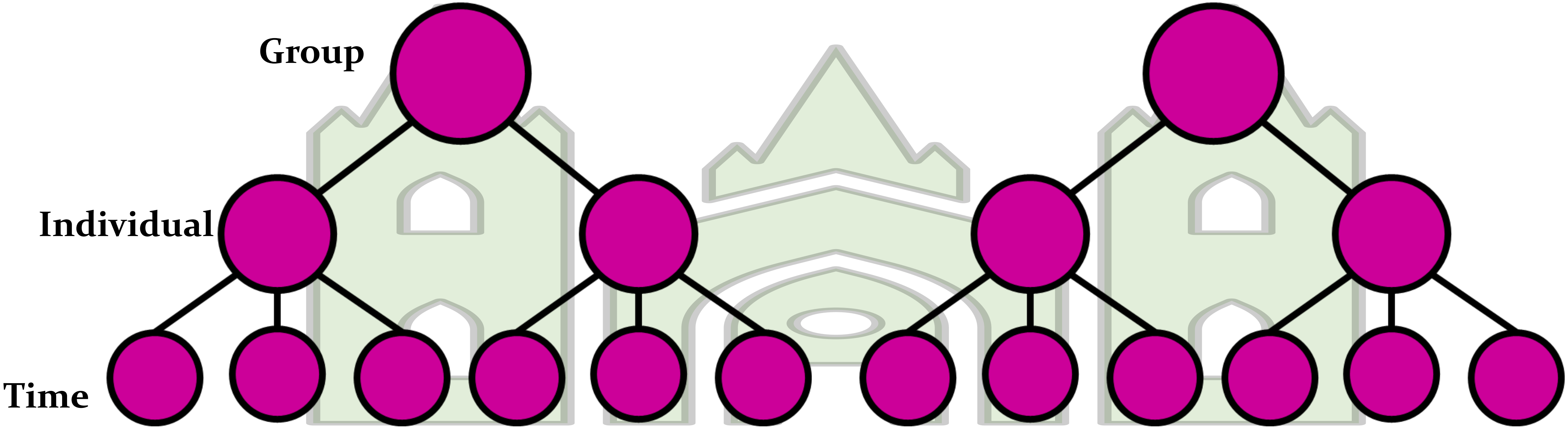

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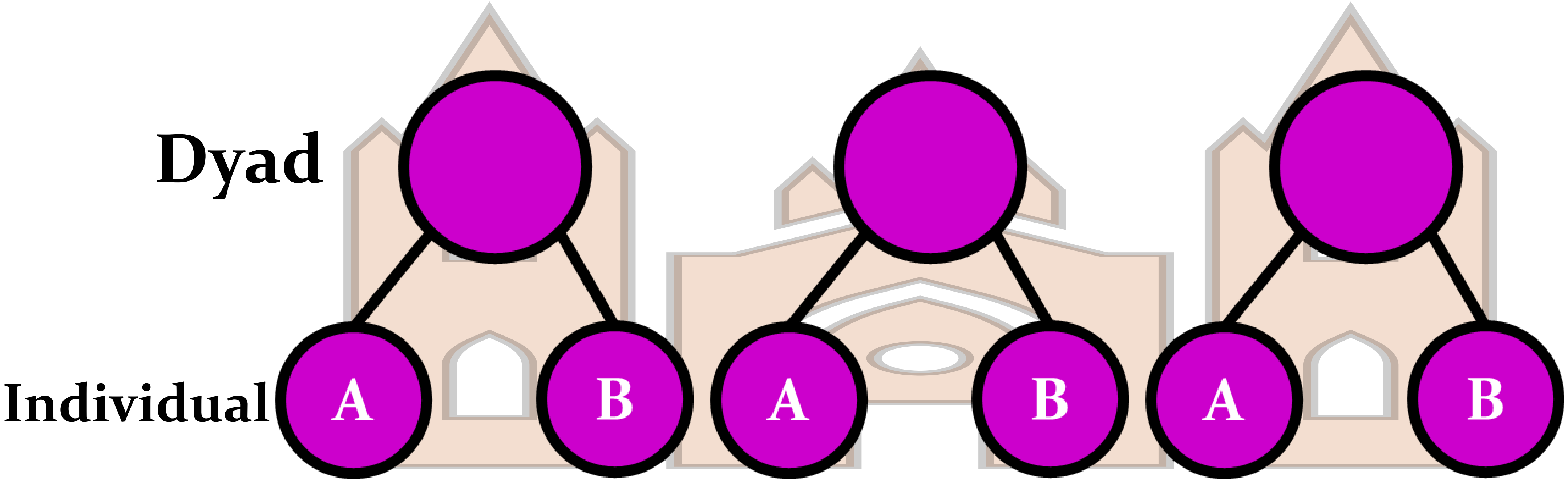

Worked Examples

R and SAS

- Example 1: Fire Damage and Knights
- Example 2: Fire Damage and Archers
- Example 3: Dragon Hits and Arrows
- Example 4: Dragon Gold and Training

Example 1: Fire Damage and Knights

Dataset Creation

- https://ademos.people.uic.edu/Chapter16.html [9]
- Parameters different across England and France
- 10 Regions (5 E, 5 F), each with knights and archers
- Knights and archers centered
- Fire damage as function of knights

	Fire_Damage	Knights	Archers	Region	Knights.C	Archers.C
1	47.09645	10.427925	8.508462	1	-0.39715575	0.6401865
2	27.62843	10.959316	11.843101	1	0.13423551	3.9748253
3	26.85106	10.175657	11.500109	1	-0.64942335	3.6318333
4	56.58726	10.887717	4.982892	1	0.06263626	-2.8853835
5	67.97403	9.274324	3.918132	1	-1.55075660	-3.9501436
6	37.26558	10.245348	10.000532	1	-0.57973270	2.1322562

Example 1: Fire Damage and Knights

Model 1: Knight Standard Regression

R

```
>D1_M1 <-lm(Fire_Damage ~ Knights.C,
data=Dragon1)

>summary(D1_M1)

>ggplot(data=Dragon1,
        aes(x=Knights.C, y=Fire_Damage)) +
        geom_point() +
        geom_smooth(method=lm, color="red")
```

SAS

```
PROC GLIMMIX data=Dragon1 method=MMPL;
        model Fire_Damage=Knights_C/s;
        output out=D1M1_pred pred lcl ucl;
PROC SORT data=D1M1_pred;
        by Knights_C;
PROC SGPLOT data=D1M1_pred noautolegend;
        band x=Knights_C lower=lcl upper=ucl;
        scatter x=Knights_C y=Fire_Damage;
        series x=Knights_C y=pred;
```

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Example 1: Fire Damage and Knights

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Model 1: Knight Standard Regression

```
Call:
lm(formula = Fire_Damage ~ Knights.C, data = Dragon1)

Residuals:
   Min      1Q  Median      3Q     Max
-35.049 -11.422  -2.464  10.182  48.416

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  42.6837     1.6140  26.445  < 2e-16 ***
Knights.C    -3.1590     0.6919  -4.565 1.45e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 16.14 on 98 degrees of freedom
Multiple R-squared:  0.1754,	Adjusted R-squared:  0.167
F-statistic: 20.84 on 1 and 98 DF,  p-value: 1.449e-05
```

Fit Statistics	
-2 Log Likelihood	838.03
AIC (smaller is better)	844.03
AICC (smaller is better)	844.28
BIC (smaller is better)	851.85
CAIC (smaller is better)	854.85
HQIC (smaller is better)	847.19
Pearson Chi-Square	25530.03
Pearson Chi-Square / DF	255.30

Parameter Estimates

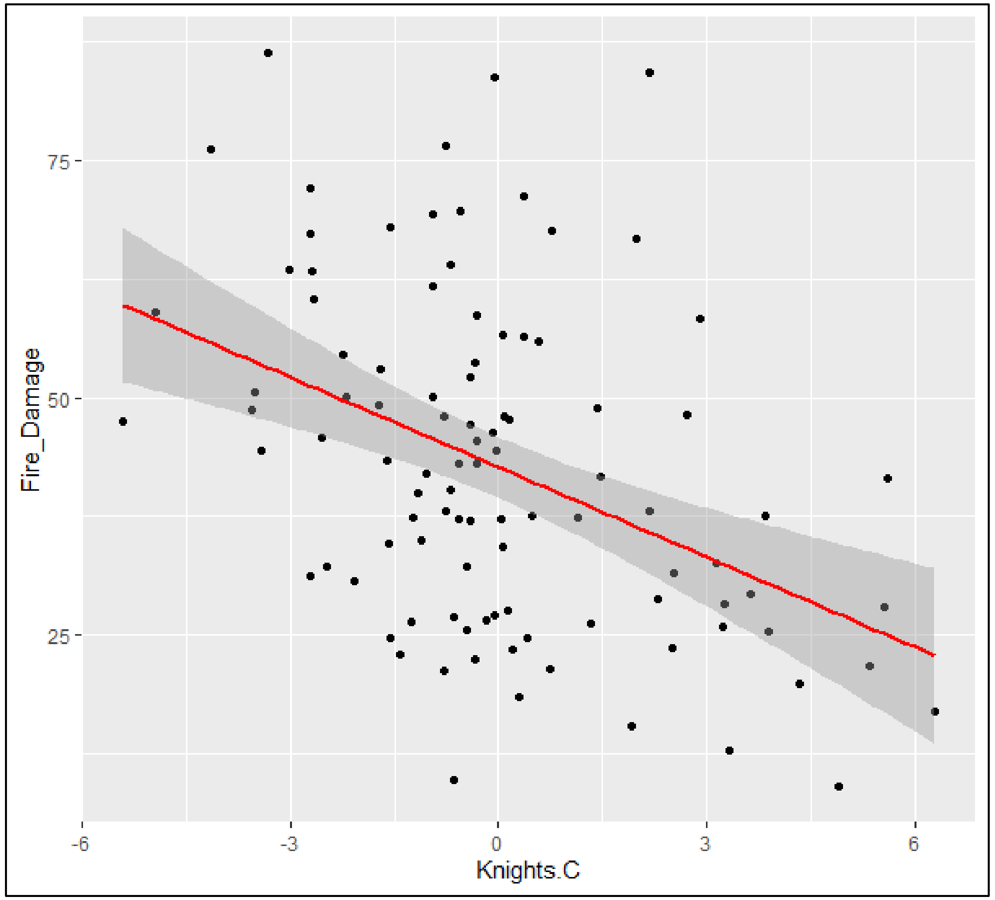
Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	42.6840	1.5978	98	26.71	<.0001
Knights_C	-3.1590	0.6850	98	-4.61	<.0001
Scale	255.30	36.1049	.	.	.

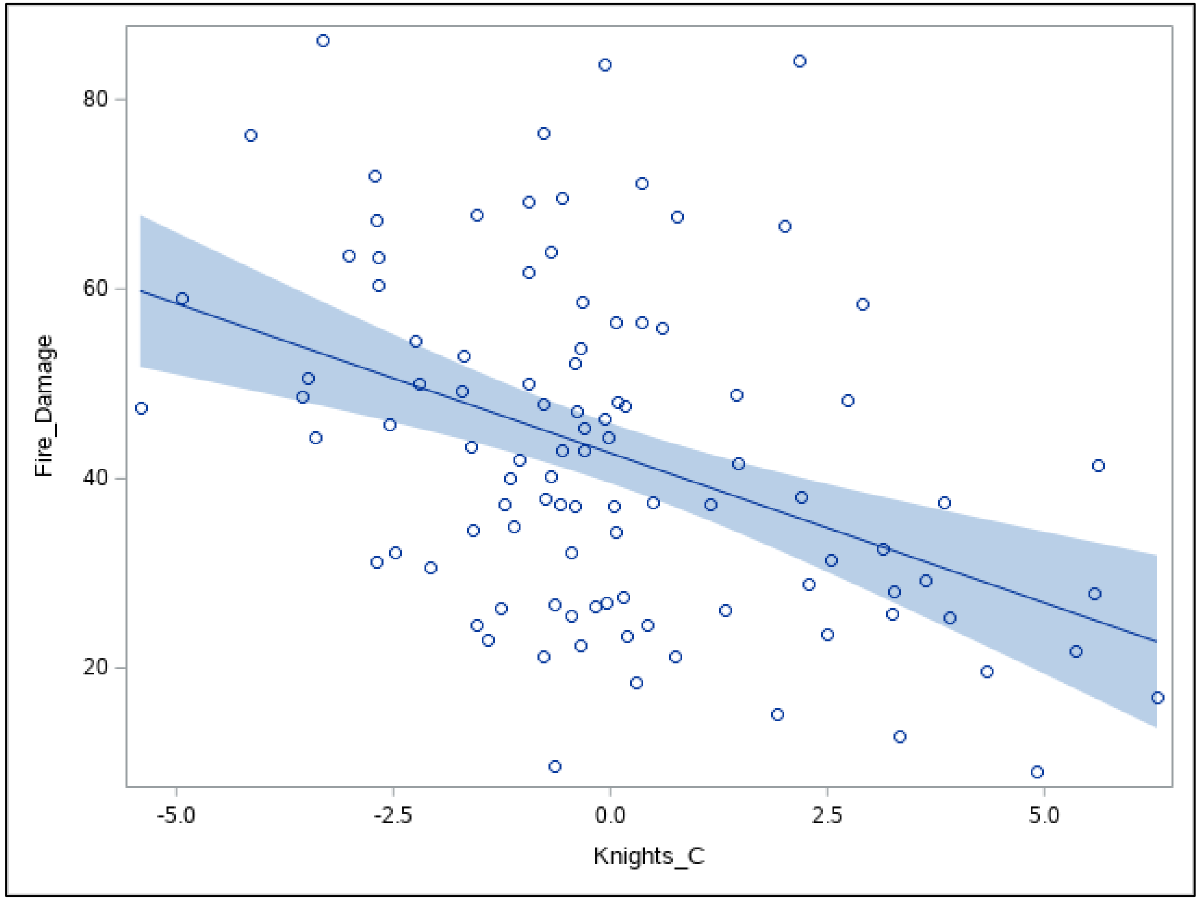
Type III Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
Knights_C	1	98	21.27	<.0001

Example 1: Fire Damage and Knights

Model 1: Knight Standard Regression





Example 1: Fire Damage and Knights

Model 2: Knight Level-2 Random Intercepts

R

```
>D1_M2 <-lmer(Fire_Damage ~ Knights.C + (1|Region), data=Dragon1, REML=F)
>summary(D1_M2)
>Dragon1$Fire_Damage_KPred <-predict(D1_M2, newdata=Dragon1)

>ggplot(data=Dragon1, aes(x=Knights.C,  y=Fire_Damage_KPred,
  group=Region)) +
  geom_point(aes(color=Region))+
  geom_smooth(method='lm', se=TRUE, aes(colour=Region))+
  xlab("Knights") + ylab("Predicted Fire Damage")+
  theme(legend.position = "none")
```

SAS

```
PROC GLIMMIX data=Dragon1 method=MMPL;
        class Region;
        model Fire_Damage=Knights_C/s;
        random Region;
        output out=D1M2_pred pred lcl ucl;
PROC SORT data=D1M2_pred;
        by Knights_C;
PROC SGPLOT data=D1M2_pred noautolegend;
        band x=Knights_C lower=lcl
upper=ucl/group=Region transparency=.90;
        scatter x=Knights_C
y=Fire_Damage/group=Region;
        series x=Knights_C y=pred/group=Region;
```

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Example 1: Fire Damage and Knights

Model 2: Knight Level-2 Random Intercepts

```
Linear mixed model fit by maximum likelihood  ['lmerMod']
Formula: Fire_Damage ~ Knights.C + (1 | Region)
   Data: Dragon1

     AIC      BIC   logLik deviance df.resid
   752.7    763.2   -372.4    744.7       96

Scaled residuals:
     Min       1Q   Median       3Q      Max
-2.88811 -0.50811 -0.01185  0.61301  2.80776

Random effects:
 Groups   Name        Variance Std.Dev.
 Region   (Intercept) 184.29   13.575
 Residual              72.38    8.508
Number of obs: 100, groups:  Region, 10

Fixed effects:
            Estimate Std. Error t value
(Intercept)  42.6837     4.3764   9.753
Knights.C    -2.6581     0.4072  -6.527

Correlation of Fixed Effects:
          (Intr)
Knights.C 0.000
```

Fit Statistics	
-2 Log Likelihood	744.74
AIC (smaller is better)	752.74
AICC (smaller is better)	753.16
BIC (smaller is better)	753.95
CAIC (smaller is better)	757.95
HQIC (smaller is better)	751.41
Generalized Chi-Square	7237.87
Gener. Chi-Square / DF	72.38

Covariance Parameter Estimates

Cov Parm	Estimate	Standard Error
Region	184.29	85.6901
Residual	72.3787	10.7900

Solutions for Fixed Effects

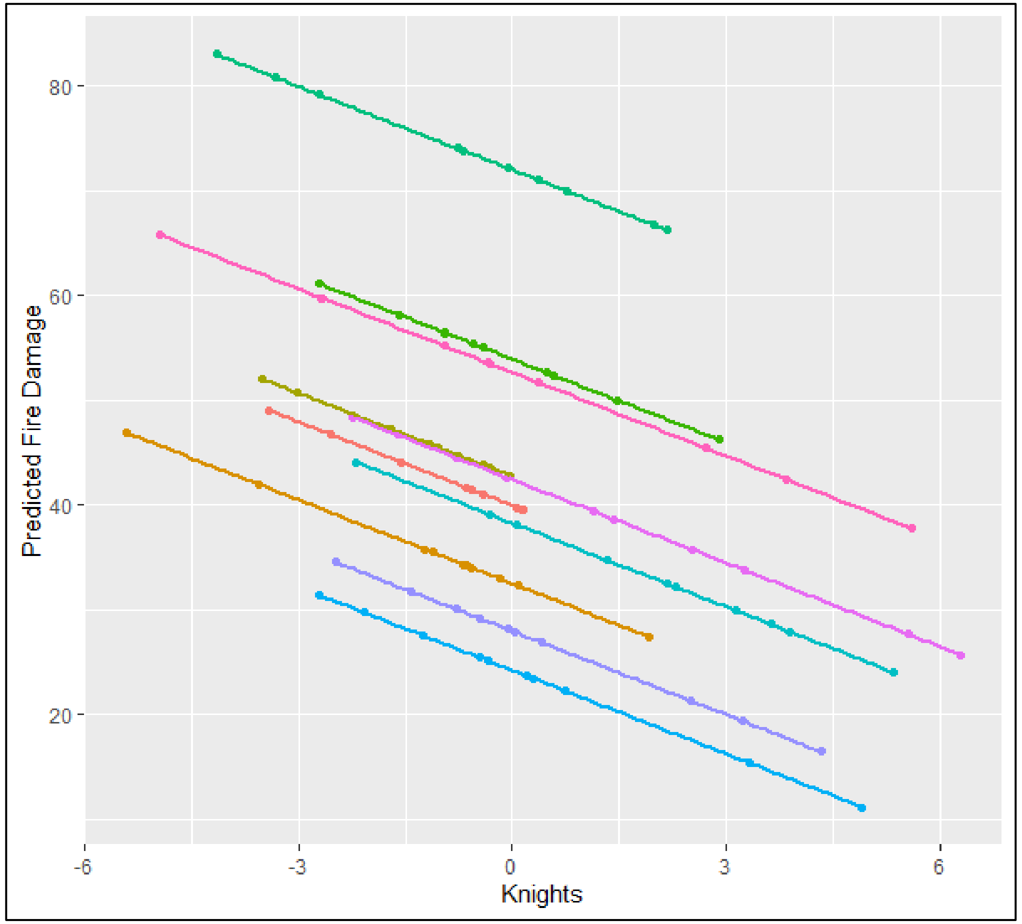
Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	42.6839	4.3763	9	9.75	<.0001
Knights_C	-2.6581	0.4072	89	-6.53	<.0001

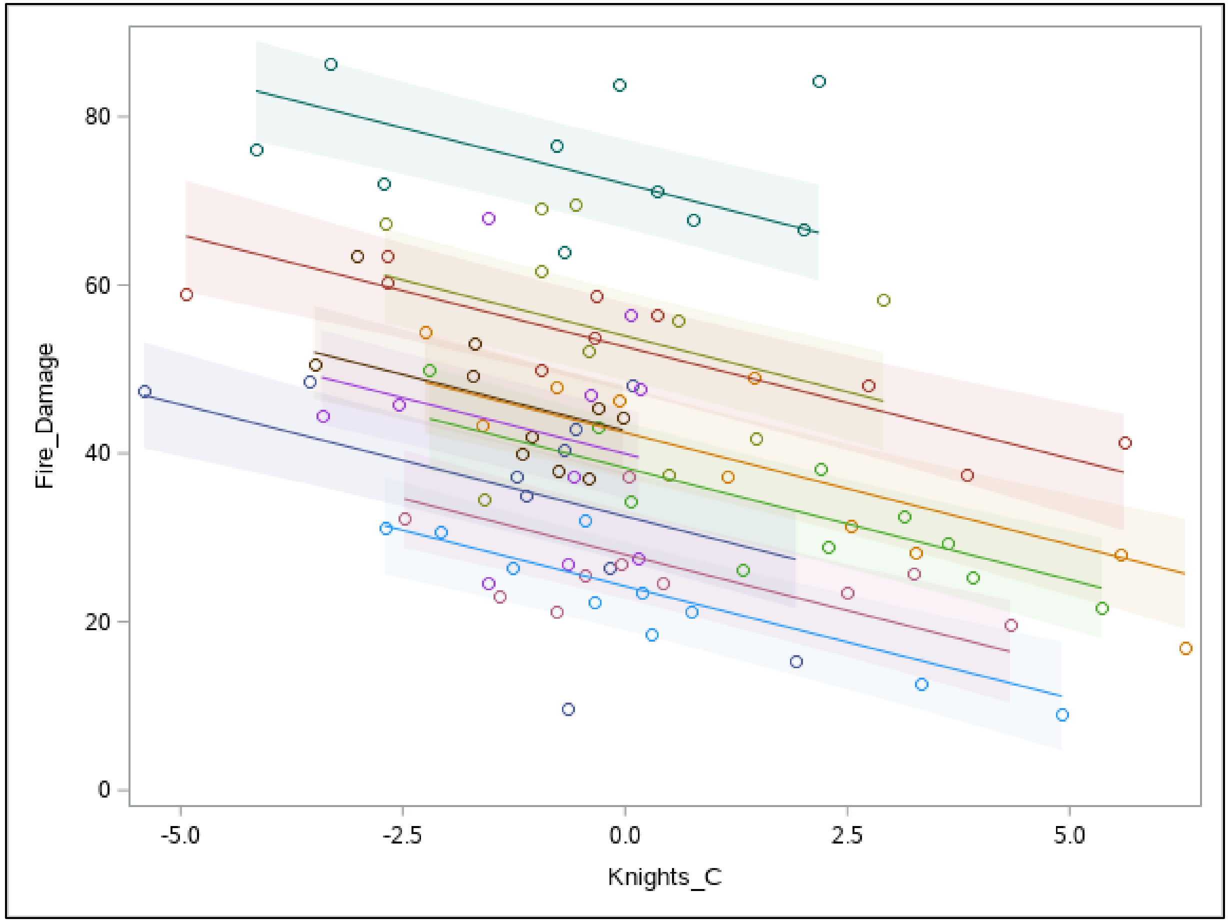
Type III Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
Knights_C	1	89	42.60	<.0001

Example 1: Fire Damage and Knights

Model 2: Knight Level-2 Random Intercepts





Example 1: Fire Damage and Knights

Model 3: Knight Level-2 Random Intercepts and Slopes

R

```
>D1_M3 <-lmer(Fire_Damage ~ Knights.C + (Knights.C|Region), data=Dragon1, REML=F)

>summary(D1_M3)

>Dragon1$Fire_Damage_KPred2 <-predict(D1_M3, newdata=Dragon1)

>ggplot(data=Dragon1, aes(x=Knights.C, y=Fire_Damage_KPred2,
  group=Region)) +
  geom_point(aes(color=Region))+
  geom_smooth(method='lm', se=TRUE, aes(colour=Region))+
  xlab("Knights") + ylab("Fire Damage")+
  theme(legend.position = "none")
```

SAS

```
PROC GLIMMIX data=Dragon1 method=MMPL;
        class Region;
        model Fire_Damage=Knights_C/s;
        random intercept Knights_C/subject = Region;
        output out=D1M3_pred pred lcl ucl;
PROC SORT data=D1M3_pred;
        by Knights_C;
PROC SGPLOT data=D1M3_pred noautolegend;
        band x=Knights_C lower=lcl
upper=ucl/group=Region transparency=.90;
        scatter x=Knights_C
y=Fire_Damage/group=Region;
        series x=Knights_C y=pred/group=Region;
```

Example 1: Fire Damage and Knights

Model 3: Knight Level-2 Random Intercepts and Slopes

```
Linear mixed model fit by maximum likelihood  ['lmerMod']
Formula: Fire_Damage ~ Knights.C + (Knights.C | Region)
   Data: Dragon1

     AIC      BIC   logLik deviance df.resid
   755.5    771.1   -371.8    743.5       94

Scaled residuals:
     Min       1Q   Median       3Q      Max
-2.87825 -0.50982 -0.06848  0.62848  2.81394

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 Region   (Intercept) 187.7426 13.7019
          Knights.C     0.2129  0.4615  1.00
 Residual              71.2306  8.4398
Number of obs: 100, groups:  Region, 10

Fixed effects:
            Estimate Std. Error t value
(Intercept)  42.7731     4.4145   9.689
Knights.C    -2.6722     0.4339  -6.159

Correlation of Fixed Effects:
          (Intr)
Knights.C 0.333
optimizer (nloptwrap) convergence code: 0 (OK)
boundary (singular) fit: see ?isSingular
```

Fit Statistics	
-2 Log Likelihood	744.74
AIC (smaller is better)	752.74
AICC (smaller is better)	753.16
BIC (smaller is better)	753.95
CAIC (smaller is better)	757.95
HQIC (smaller is better)	751.41
Generalized Chi-Square	7237.87
Gener. Chi-Square / DF	72.38

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error
Intercept	Region	184.29	85.6906
Knights_C	Region	0	.
Residual		72.3787	10.7900

Solutions for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	42.6839	4.3764	9	9.75	<.0001
Knights_C	-2.6581	0.4072	9	-6.53	0.0001

Type III Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
Knights_C	1	9	42.60	0.0001

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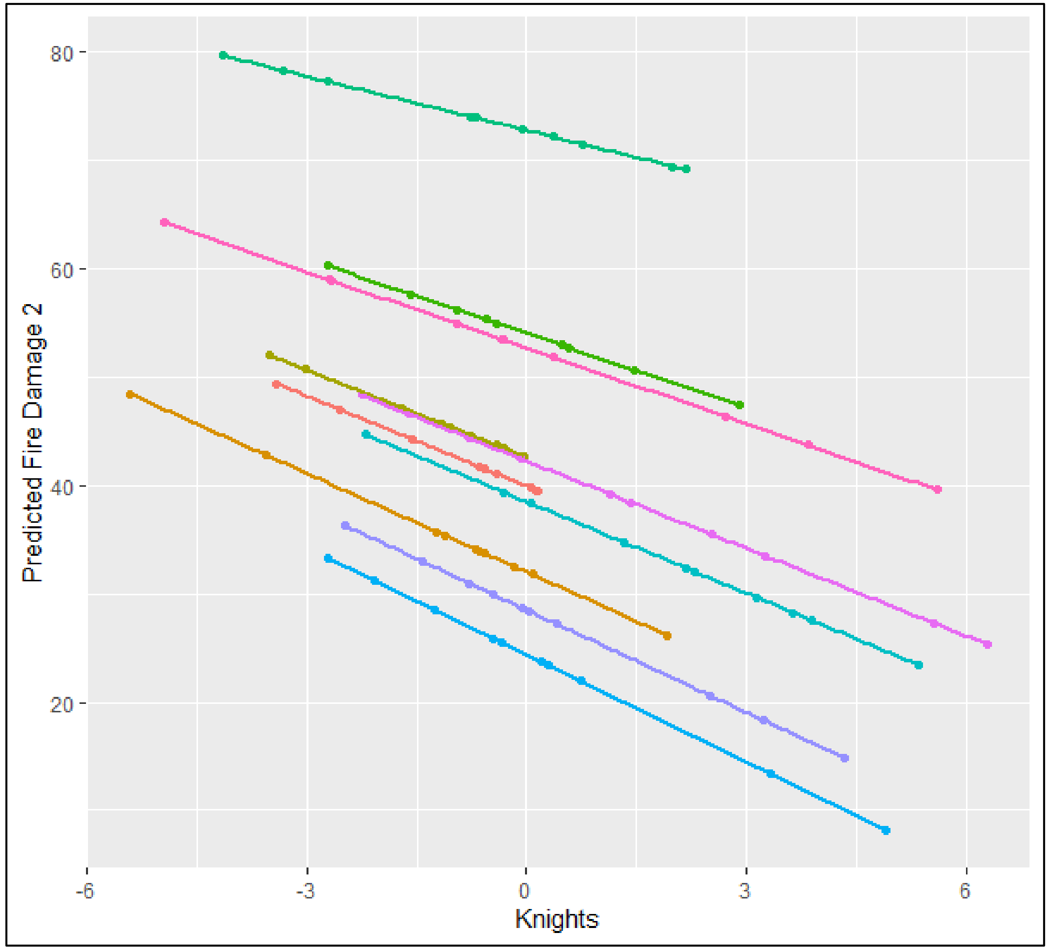
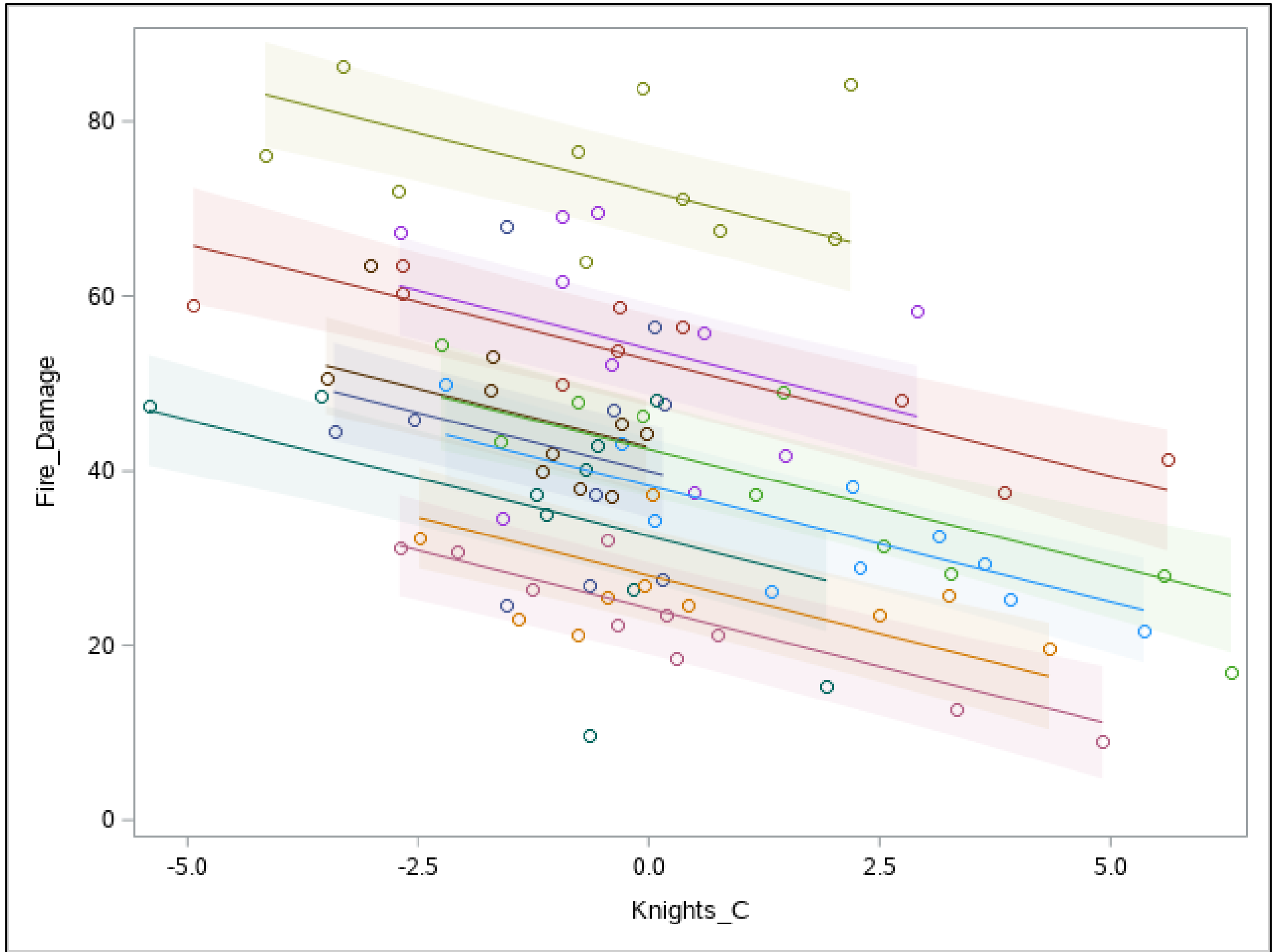
DAKOTA COMMUNITY COLLABORATIVE ON TRANSLATIONAL ACTIVITY

Example 1: Fire Damage and Knights

UNIVERSITY OF NORTH DAKOTA

Model 3: Knight Level-2 Random Intercepts and Slopes





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DAKOTA COMMUNITY COLLABORATIVE ON TRANSLATIONAL ACTIVITY

Example 2: Fire Damage and Archers

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Model 1: Archer Standard Regression

R

```
>D2_M1 <-lm(Fire_Damage ~ Archers.C, data=Dragon1)
>summary(D2_M1)


>ggplot(data=Dragon1,
        aes(x=Archers.C, y=Fire_Damage)) +
        geom_point() +geom_smooth(method=lm, color="red")
```

SAS

```
PROC GLIMMIX data=Dragon1 method=MMPL;
        model Fire_Damage=Archers_C/s;
        output out=D2M1_pred pred lcl ucl;
PROC SORT data=D2M1_pred;
        by Archers_C;
PROC SGPLOT data=D2M1_pred noautolegend;
        band x=Archers_C lower=lcl upper=ucl;
        scatter x=Archers_C y=Fire_Damage;
        series x=Archers_C y=pred;
```

Example 2: Fire Damage and Archers

Model 1: Archer Standard Regression

```
Call:
lm(formula = Fire_Damage ~ Archers.C, data = Dragon1)

Residuals:
    Min      1Q  Median      3Q     Max
-41.077 -12.787  -2.217  11.727  43.842

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  42.6837     1.7361  24.586   <2e-16 ***
Archers.C     1.4808     0.6818   2.172   0.0323 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 17.36 on 98 degrees of freedom
Multiple R-squared:  0.04592,	Adjusted R-squared:  0.03619
F-statistic: 4.717 on 1 and 98 DF,  p-value: 0.03227
```

Fit Statistics	
-2 Log Likelihood	852.61
AIC (smaller is better)	858.61
AICC (smaller is better)	858.86
BIC (smaller is better)	866.43
CAIC (smaller is better)	869.43
HQIC (smaller is better)	861.78
Pearson Chi-Square	29538.20
Pearson Chi-Square / DF	295.38

Parameter Estimates

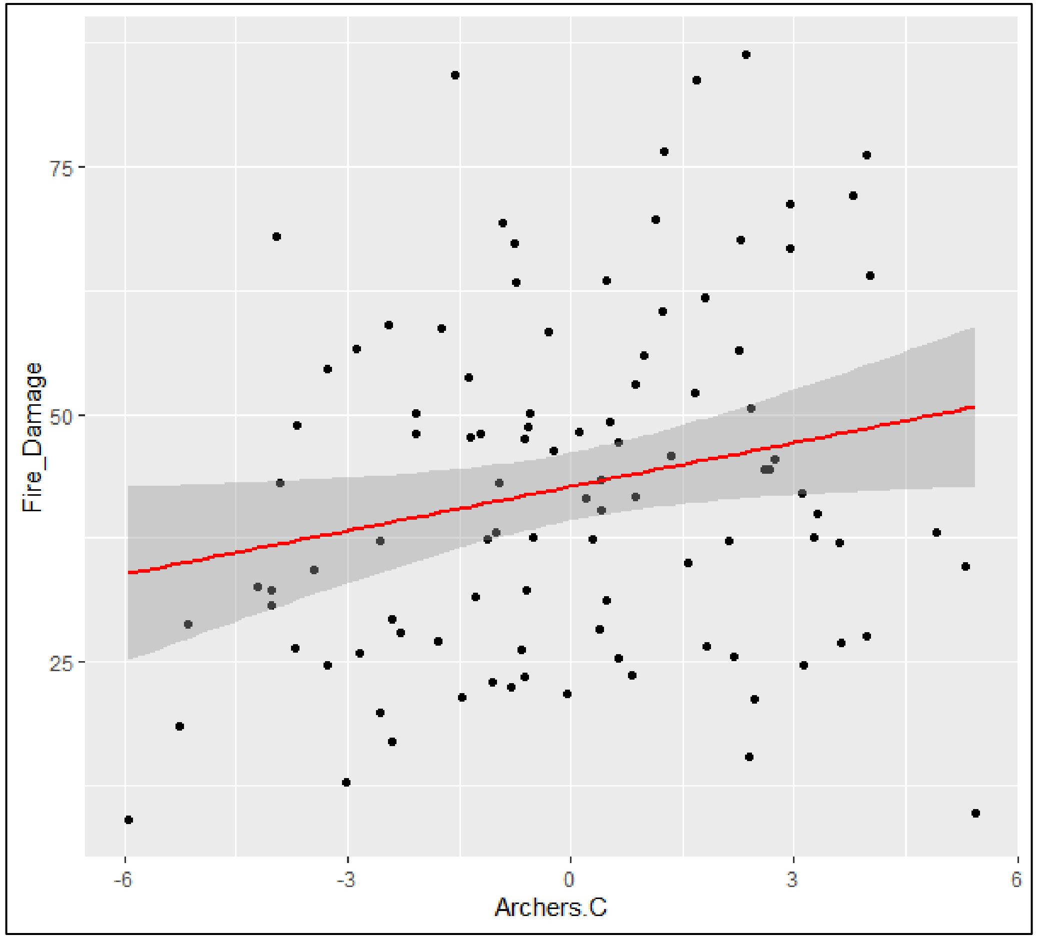
Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	42.6833	1.7187	98	24.84	<.0001
Archers_C	1.4808	0.6749	98	2.19	0.0306
Scale	295.38	41.7733	.	.	.

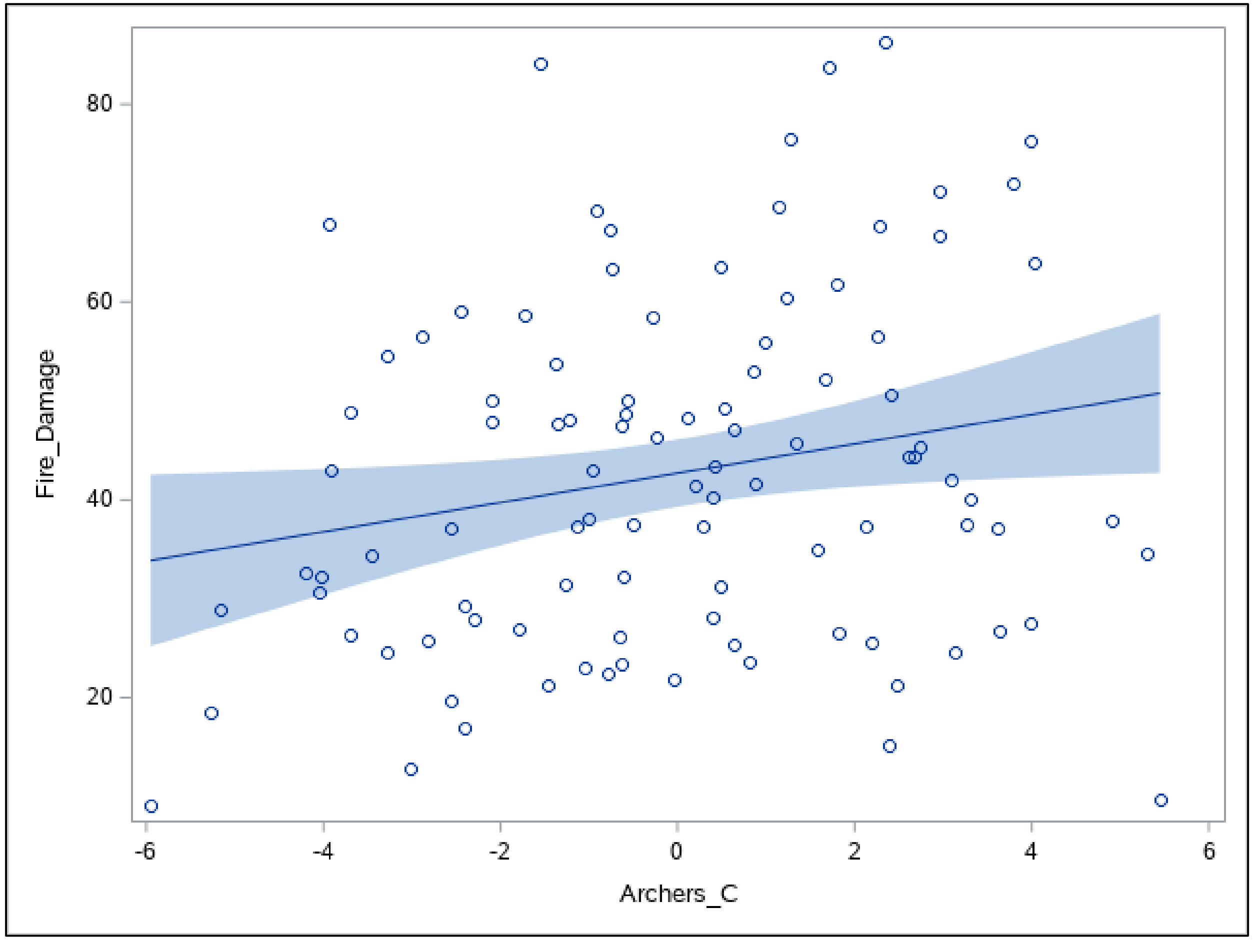
Type III Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
Archers_C	1	98	4.81	0.0306

Example 2: Fire Damage and Archers

Model 1: Archer Standard Regression





Example 2: Fire Damage and Archers

Model 2: Archer Level-2 Random Intercepts

R

```
>D2_M2 <-lmer(Fire_Damage ~ Archers.C + (1|Region), data=Dragon1, REML=F)
>summary(D2_M2)

>Dragon1$Fire_Damage_APred <-predict(D2_M2, newdata=Dragon1)

>ggplot(data=Dragon1, aes(x=Archers.C, y=Fire_Damage_APred,
  group=Region)) +
  geom_point(aes(color=Region))+
  geom_smooth(method='lm', se=TRUE, aes(colour=Region))+
  xlab("Archers") + ylab("Fire Damage")+
  theme(legend.position = "none")
```

SAS

```
PROC GLIMMIX data=Dragon1 method=MMPL;
        class Region;
        model Fire_Damage=Archers_C/s;
        random Region;
        output out=D2M2_pred pred lcl ucl;
PROC SORT data=D2M2_pred;
        by Knights_C;
PROC SGPLOT data=D2M2_pred noautolegend;
        band x=Archers_C lower=lcl
upper=ucl/group=Region transparency=.90;
        scatter x=Archers_C
y=Fire_Damage/group=Region;
        series x=Archers_C y=pred/group=Region;
```

Example 2: Fire Damage and Archers

Model 2: Archer Level-2 Random Intercepts

```
Linear mixed model fit by maximum likelihood  ['lmerMod']
Formula: Fire_Damage ~ Archers.C + (1 | Region)
   Data: Dragon1

     AIC      BIC   logLik deviance df.resid
   762.3    772.7   -377.1    754.3       96

Scaled residuals:
     Min       1Q   Median       3Q      Max
-2.74765 -0.65590  0.00371  0.73893  1.96885

Random effects:
 Groups   Name        Variance Std.Dev.
 Region   (Intercept) 331.68   18.212
 Residual              75.53    8.691
Number of obs: 100, groups:  Region, 10

Fixed effects:
            Estimate Std. Error t value
(Intercept)  42.6837     5.8244   7.328
Archers.C    -2.6720     0.4705  -5.679

Correlation of Fixed Effects:
          (Intr)
Archers.C 0.000
```

Fit Statistics	
-2 Log Likelihood	754.29
AIC (smaller is better)	762.29
AICC (smaller is better)	762.71
BIC (smaller is better)	763.50
CAIC (smaller is better)	767.50
HQIC (smaller is better)	760.96
Generalized Chi-Square	7552.79
Gener. Chi-Square / DF	75.53

Covariance Parameter Estimates

Cov Parm	Estimate	Standard Error
Region	331.68	154.00
Residual	75.5279	11.2779

Solutions for Fixed Effects

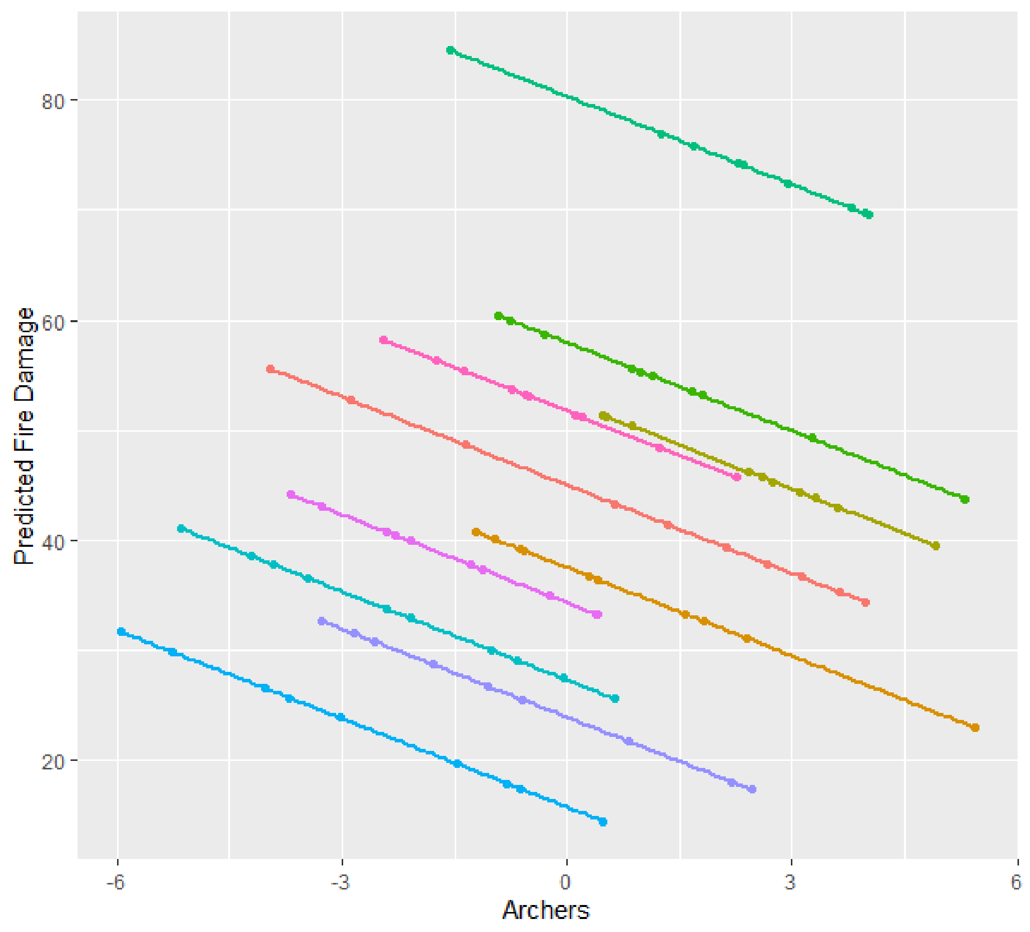
Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	42.6845	5.8244	9	7.33	<.0001
Archers_C	-2.6720	0.4705	89	-5.68	<.0001

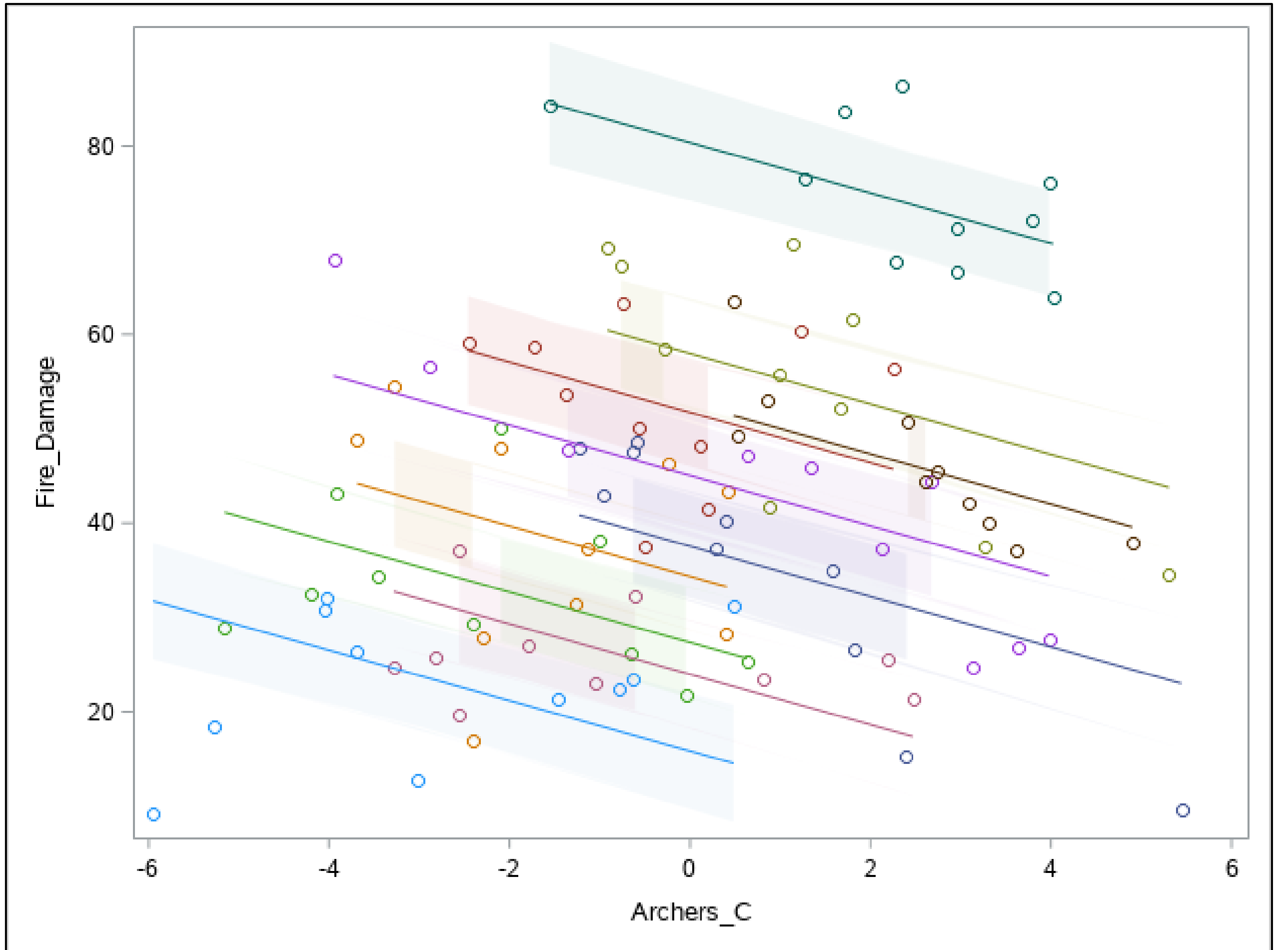
Type III Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
Archers_C	1	89	32.25	<.0001

Example 2: Fire Damage and Archers

Model 2: Archer Level-2 Random Intercepts





Example 2: Fire Damage and Archers

Model 3: Archer Level-2 Random Intercepts and Slopes

R

```
>D2_M3 <-lmer(Fire_Damage ~ Archers.C + (Archers.C|Region), data=Dragon1, REML=F)
>summary(D2_M3)

>Dragon1$Fire_Damage_APred2 <-predict(D2_M3, newdata=Dragon1)

>ggplot(data=Dragon1, aes(x=Archers.C, y=Fire_Damage_APred2,
  group=Region)) +
  geom_point(aes(color=Region))+
  geom_smooth(method='lm', se=TRUE, aes(colour=Region))+
  xlab("Archers") + ylab("Fire Damage")+
  theme(legend.position = "none")
```

SAS

```
PROC GLIMMIX data=Dragon1 method=MMPL;
	class Region;
	model Fire_Damage=Archers_C/s;
	random intercept Archers/subject = Region;
	output out=D2M3_pred pred lcl ucl;
PROC SORT data=D2M3_pred;
	by Archers_C;
PROC SGPLOT data=D2M3_pred noautolegend;
	band x=Archers_C lower=lcl
upper=ucl/group=Region transparency=.90;
	scatter x=Archers_C
y=Fire_Damage/group=Region;
	series x=Archers_C y=pred/group=Region;
```

Example 2: Fire Damage and Archers

Model 3: Archer Level-2 Random Intercepts and Slopes

```
Linear mixed model fit by maximum likelihood  ['lmerMod']
Formula: Fire_Damage ~ Archers.C + (Archers.C | Region)
   Data: Dragon1

     AIC      BIC   logLik deviance df.resid
   752.1    767.7   -370.0    740.1       94

Scaled residuals:
    Min      1Q  Median      3Q     Max
-3.0279 -0.4983 -0.1237  0.6095  2.2319

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 Region   (Intercept) 297.280  17.242
          Archers.C     4.398   2.097   -0.50
 Residual              58.172   7.627
Number of obs: 100, groups:  Region, 10

Fixed effects:
            Estimate Std. Error t value
(Intercept)  45.2840     5.5414   8.172
Archers.C    -2.6568     0.7941  -3.346

Correlation of Fixed Effects:
          (Intr)
Archers.C -0.418
```

Fit Statistics	
-2 Log Likelihood	742.01
AIC (smaller is better)	752.01
AICC (smaller is better)	752.65
BIC (smaller is better)	753.52
CAIC (smaller is better)	758.52
HQIC (smaller is better)	750.35
Generalized Chi-Square	5849.44
Gener. Chi-Square / DF	58.49

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error
Intercept	Region	282.39	132.55
Archers_C	Region	4.4534	2.7237
Residual		58.4944	9.1638

Solutions for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	44.9369	5.4090	9	8.31	<.0001
Archers_C	-2.5597	0.7991	9	-3.20	0.0108

Type III Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
Archers_C	1	9	10.26	0.0108

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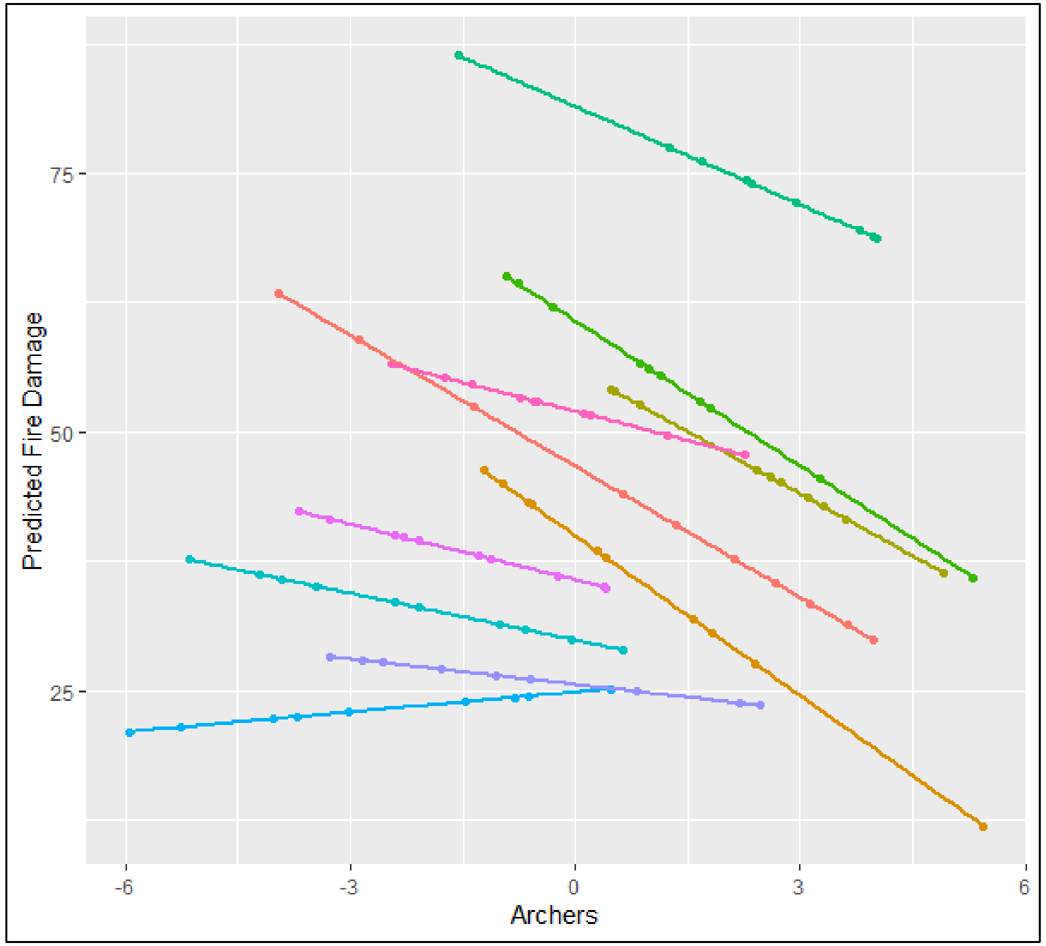
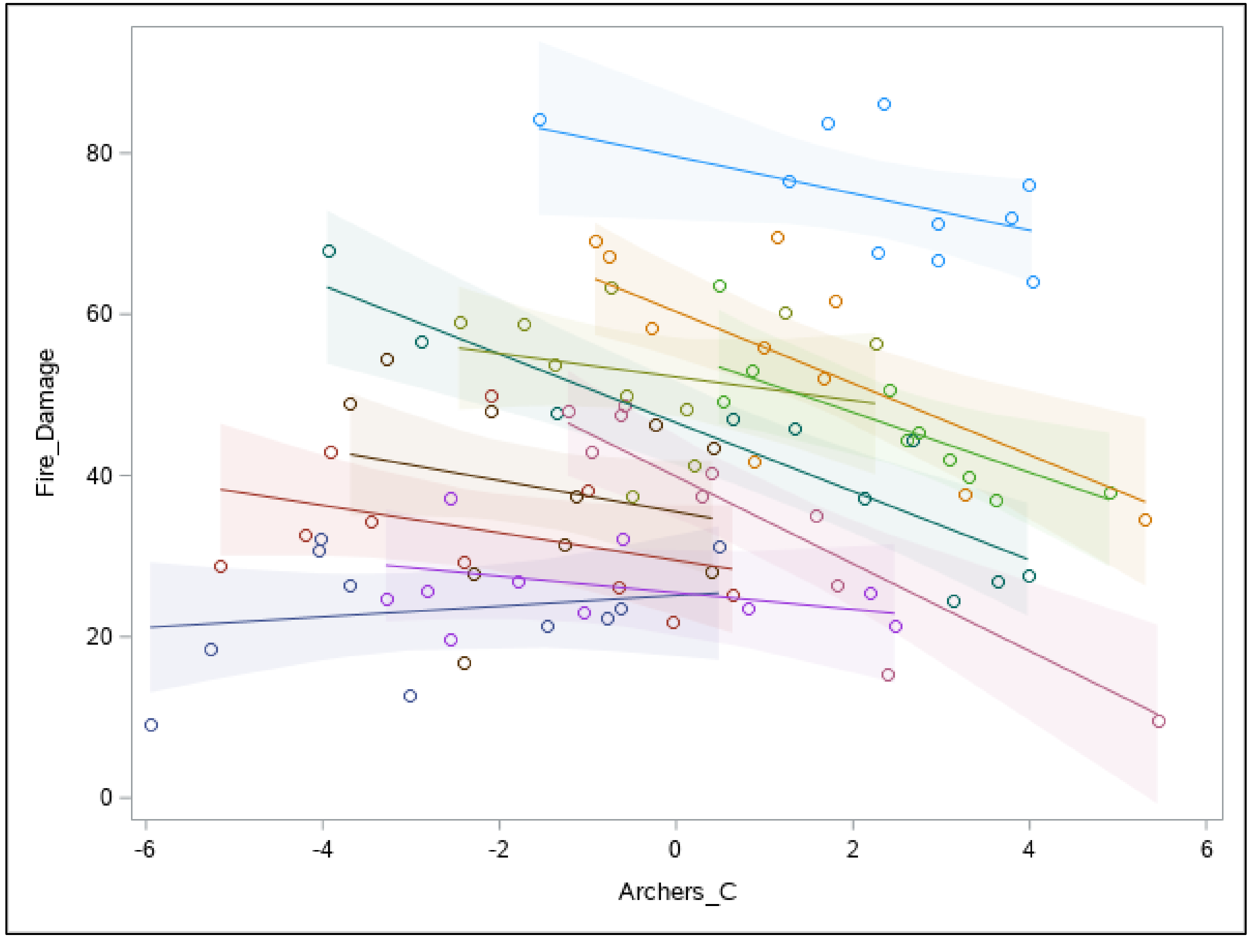
Example 2: Fire Damage and Archers

Model 3: Archer Level-2 Random Intercepts and Slopes





Example 3: Dragon Hits and Arrows

Dataset Creation

- https://data.princeton.edu/pop510/lang2 [10]
- Got data and transformed it to fit the dragon theme

```
>testds <- read.dta("https://data.princeton.edu/pop510/snijders.dta")
>testds2 <-testds[testds$schoolnr<100,] #Subset
   #langpost <-DragonHit
   #iavc  <-ArrowNumber
   #schoolnr <-Cathedral
>Dragon2 <-data.frame("DragonHit"=testds2$langpost/5,
         "Arrows.C"=testds2$iq_verb*5-mean(testds2$iq_verb*5),
         "Cathedral"=testds2$schoolnr)
>head(Dragon2)
```

```
  DragonHit   Arrows.C Cathedral
1       9.2  16.056511         1
2       9.0  13.556511         1
3       6.6 -11.443489         1
4       9.2  -3.943489         1
5       4.0 -18.943489         1
6       6.0 -11.443489         1
```

Example 3: Dragon Hits and Arrows

Model 1: Arrow Standard Regression

R

```
>D3_M1 <- lm(DragonHit ~ Arrows.C,
        data=Dragon2)
>summary(D3_M1)


>ggplot(data=Dragon2, aes(x=Arrows.C,
        y=DragonHit)) +
        geom_point() +
        geom_smooth(method=lm, color="red")
```

SAS

```
PROC GLIMMIX data=Dragon2 method=MMPL;
        model DragonHit=Arrows_C/s;
        output out=D3M1_pred pred lcl ucl;
PROC SORT data=D3M1_pred;
        by Arrows_C;
PROC SGPLOT data=D3M1_pred noautolegend;
        band x=Arrows_C lower=lcl upper=ucl;
        scatter x=Arrows_C y=DragonHit;
        series x=Arrows_C y=pred;
```

Example 3: Dragon Hits and Arrows

Model 1: Arrow Standard Regression

```
Call:
lm(formula = DragonHit ~ Arrows.C, data = Dragon2)

Residuals:
    Min      1Q  Median      3Q     Max
-4.5474 -0.9384  0.0877  1.0616  3.2003

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 7.874939   0.050464  156.05   <2e-16 ***
Arrows.C    0.109550   0.004652   23.55   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.44 on 812 degrees of freedom
Multiple R-squared:  0.4058,	Adjusted R-squared:  0.4051
F-statistic: 554.6 on 1 and 812 DF,  p-value: < 2.2e-16
```

Fit Statistics	
-2 Log Likelihood	2901.40
AIC (smaller is better)	2907.40
AICC (smaller is better)	2907.43
BIC (smaller is better)	2921.51
CAIC (smaller is better)	2924.51
HQIC (smaller is better)	2912.82
Pearson Chi-Square	1683.22
Pearson Chi-Square / DF	2.07

Parameter Estimates

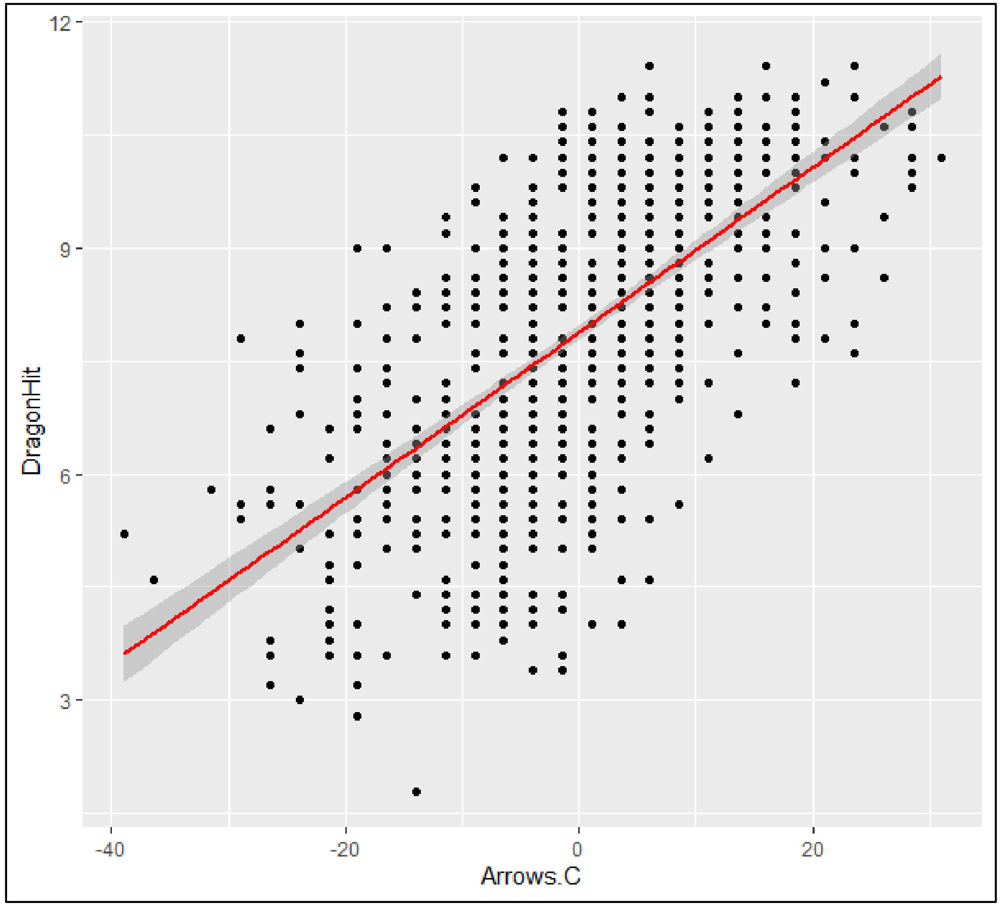
Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	7.8749	0.05040	812	156.24	<.0001
Arrows_C	0.1095	0.004646	812	23.58	<.0001
Scale	2.0678	0.1025	.	.	.

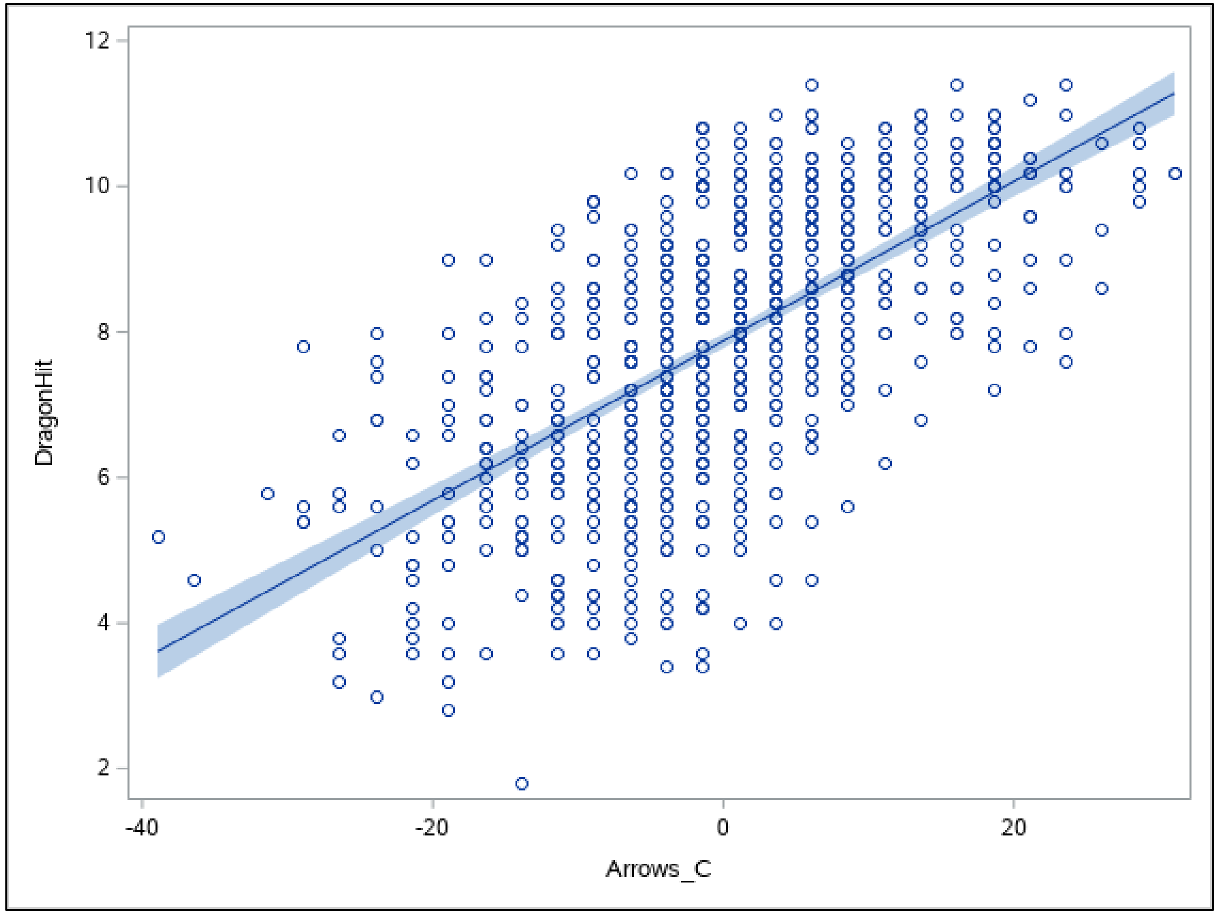
Type III Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
Arrows_C	1	812	555.95	<.0001

Example 3: Dragon Hits and Arrows

Model 1: Arrow Standard Regression





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Example 3: Dragon Hits and Arrows

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Model 2: Arrow Level-2 Random Intercepts

R

```
>D3_M2 <- lmer(DragonHit ~ Arrows.C + (1|Cathedral), data=Dragon2, REML=F)
>summary(D3_M2)

>Dragon2$DragonHit_Pred <-predict(D3_M2, newdata=Dragon2)

>ggplot(data=Dragon2, aes(x=Arrows.C, y=DragonHit_Pred,
  group=Cathedral)) +
  geom_point(aes(color=Cathedral))+
  geom_smooth(method='lm', se=F, aes(colour=Cathedral))+
  xlab("Arrows") + ylab("Dragon Hits")+
  theme(legend.position = "none")
```

SAS

```
PROC GLIMMIX data=Dragon2 method=MMPL;
        class Cathedral;
        model DragonHit=Arrows_C/s;
        random Cathedral;
        output out=D3M2_pred pred lcl ucl;
PROC SORT data=D3M2_pred;
        by Arrows_C;
PROC SGPLOT data=D3M2_pred noautolegend;
        band x=Arrows_C lower=lcl
upper=ucl/group=Cathedral transparency=.90;
        scatter x=Arrows_C y=DragonHit;
        series x=Arrows_C y=pred/group=Cathedral;
```

Example 3: Dragon Hits and Arrows

Model 2: Arrow Level-2 Random Intercepts

```
Linear mixed model fit by maximum likelihood  ['lmerMod']
Formula: DragonHit ~ Arrows.C + (1 | Cathedral)
   Data: Dragon2

     AIC      BIC   logLik deviance df.resid
  2839.4   2858.2  -1415.7   2831.4      810

Scaled residuals:
    Min      1Q  Median      3Q     Max
-3.2753 -0.6202  0.0361  0.7280  2.1384

Random effects:
 Groups    Name        Variance Std.Dev.
 Cathedral (Intercept) 0.3328   0.5769
 Residual              1.7507   1.3231
Number of obs: 814, groups:  Cathedral, 46

Fixed effects:
            Estimate Std. Error t value
(Intercept) 7.818114   0.098527   79.35
Arrows.C    0.103365   0.004584   22.55

Correlation of Fixed Effects:
         (Intr)
Arrows.C 0.023
```

Fit Statistics	
-2 Log Likelihood	2831.38
AIC (smaller is better)	2839.38
AICC (smaller is better)	2839.43
BIC (smaller is better)	2846.69
CAIC (smaller is better)	2850.69
HQIC (smaller is better)	2842.12
Generalized Chi-Square	1425.08
Gener. Chi-Square / DF	1.75

Covariance Parameter Estimates

Cov Parm	Estimate	Standard Error
Cathedral	0.3328	0.09537
Residual	1.7507	0.08946

Solutions for Fixed Effects

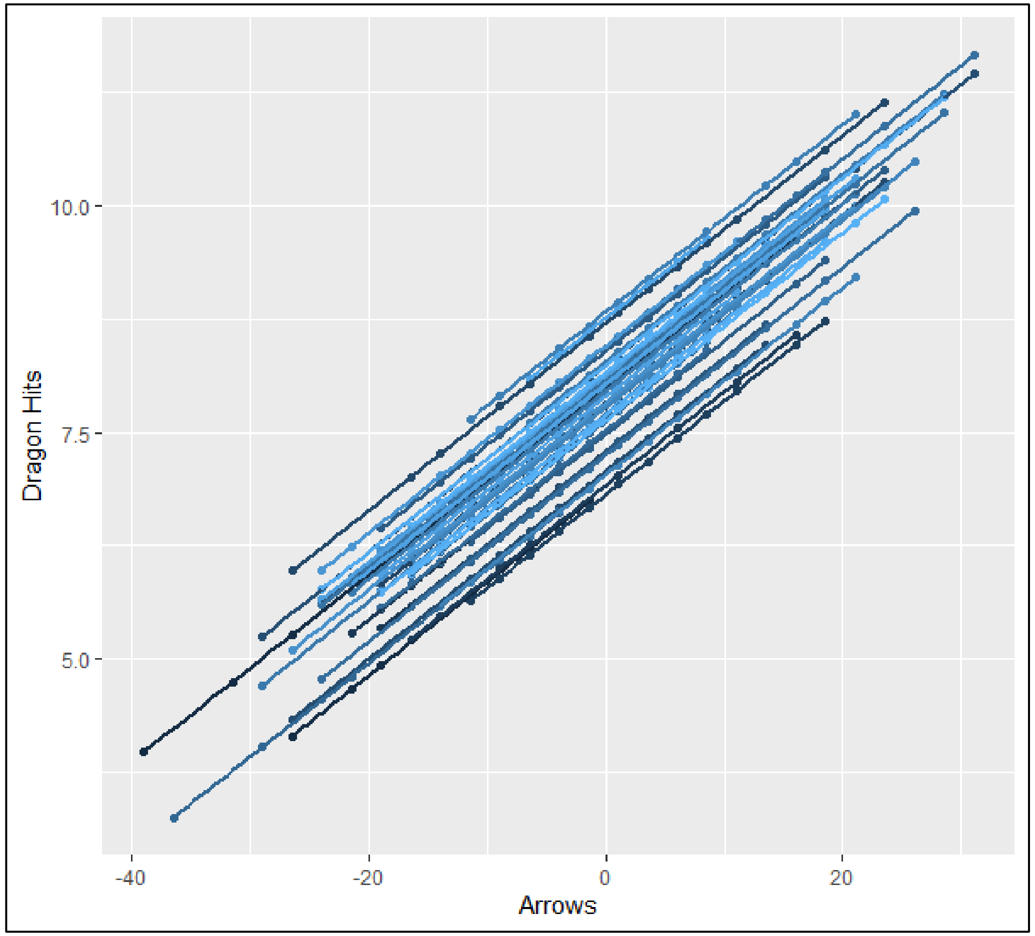
Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	7.8181	0.09853	45	79.35	<.0001
Arrows_C	0.1034	0.004584	767	22.55	<.0001

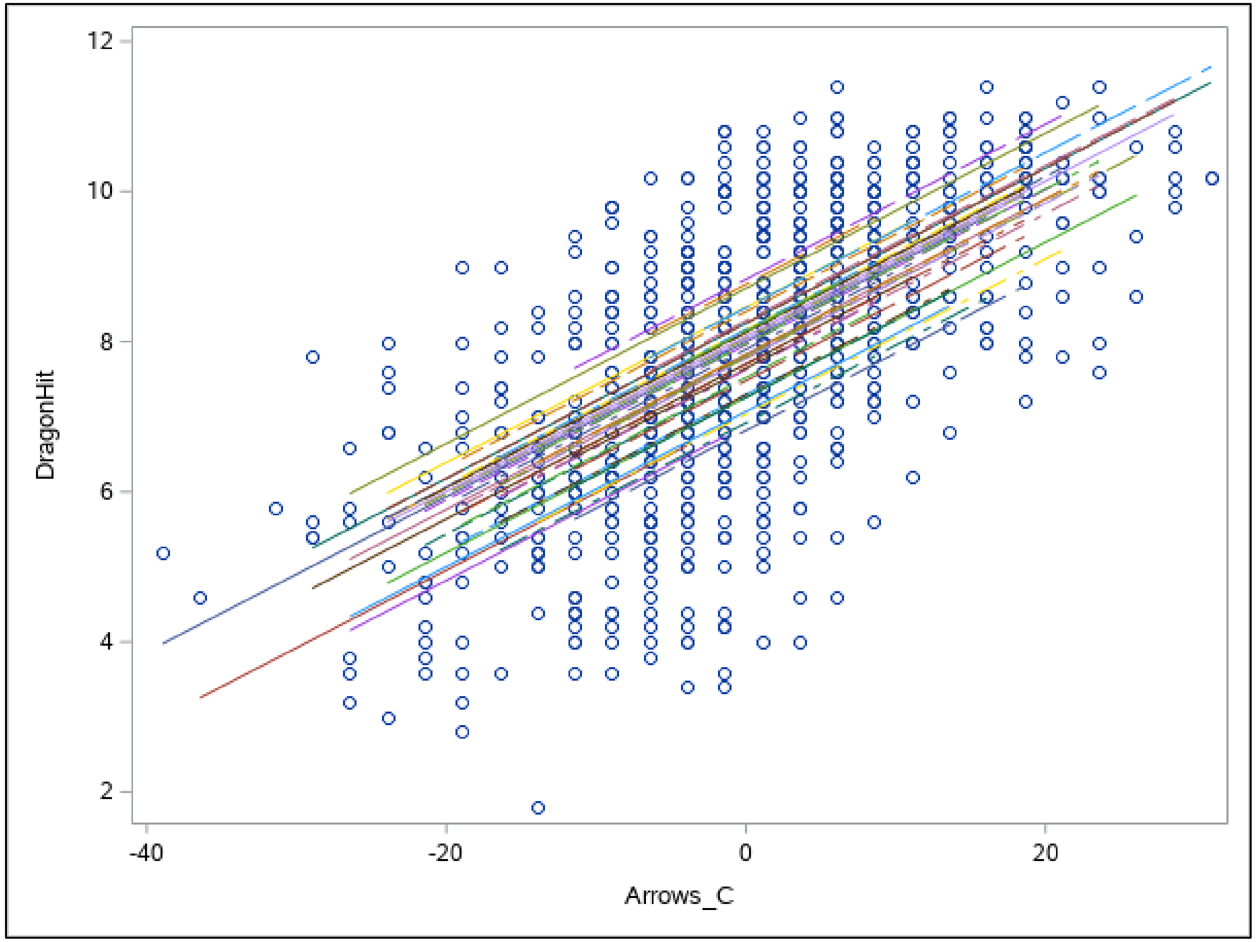
Type III Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
Arrows_C	1	767	508.49	<.0001

Example 3: Dragon Hits and Arrows

Model 2: Arrow Level-2 Random Intercepts





Example 3: Dragon Hits and Arrows

Model 3: Arrow Level-2 Random Intercepts and Slopes

R

```
>D3_M3 <- lmer(DragonHit ~ Arrows.C + (Arrows.C|Cathedral), data=Dragon2, REML=F)
summary(D3_M3)

>Dragon2$DragonHit_Pred2 <-predict(D3_M3, newdata=Dragon2)

>ggplot(data=Dragon2, aes(x=Arrows.C, y=DragonHit_Pred2,
 group=Cathedral)) +
 geom_point(aes(color=Cathedral))+
 geom_smooth(method='lm', se=F, aes(colour=Cathedral))+
 xlab("Arrows") + ylab("Dragon Hits")+
 theme(legend.position = "none")
```

SAS

```
PROC GLIMMIX data=Dragon2 method=MMPL;
        class Cathedral;
        model DragonHit=Arrows_C/s;
        random intercept Arrows_C/subject=Cathedral;
        output out=D3M3_pred pred lcl ucl;
PROC SORT data=D3M3_pred;
        by Arrows_C;
PROC SGPLOT data=D3M3_pred noautolegend;
        band x=Arrows_C lower=lcl
upper=ucl/group=Cathedral transparency=.90;
        scatter x=Arrows_C y=DragonHit;
        series x=Arrows_C y=pred/group=Cathedral;
```

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Example 3: Dragon Hits and Arrows

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Model 3: Arrow Level-2 Random Intercepts and Slopes

```
Linear mixed model fit by maximum likelihood  ['lmerMod']
Formula: DragonHit ~ Arrows.C + (Arrows.C | Cathedral)
   Data: Dragon2

     AIC      BIC   logLik deviance df.resid
  2838.5   2866.7  -1413.2   2826.5      808

Scaled residuals:
    Min      1Q  Median      3Q     Max
-3.2454 -0.6294  0.0523  0.7400  2.3003

Random effects:
 Groups    Name        Variance  Std.Dev. Corr
 Cathedral (Intercept) 0.3354974 0.57922
           Arrows.C    0.0001697 0.01303  -1.00
 Residual              1.7340747 1.31684
Number of obs: 814, groups:  Cathedral, 46

Fixed effects:
            Estimate Std. Error t value
(Intercept)  7.83793    0.09843   79.62
Arrows.C     0.10428    0.00492   21.20

Correlation of Fixed Effects:
         (Intr)
Arrows.C -0.345
optimizer (nloptwrap) convergence code: 0 (OK)
boundary (singular) fit: see ?isSingular
```

Fit Statistics

-2 Log Likelihood	2831.33
AIC (smaller is better)	2841.33
AICC (smaller is better)	2841.40
BIC (smaller is better)	2850.47
CAIC (smaller is better)	2855.47
HQIC (smaller is better)	2844.75
Generalized Chi-Square	1420.78
Gener. Chi-Square / DF	1.75

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error
Intercept	Cathedral	0.3347	0.09641
Arrows_C	Cathedral	0.000045	0.000213
Residual		1.7454	0.09224

Solutions for Fixed Effects

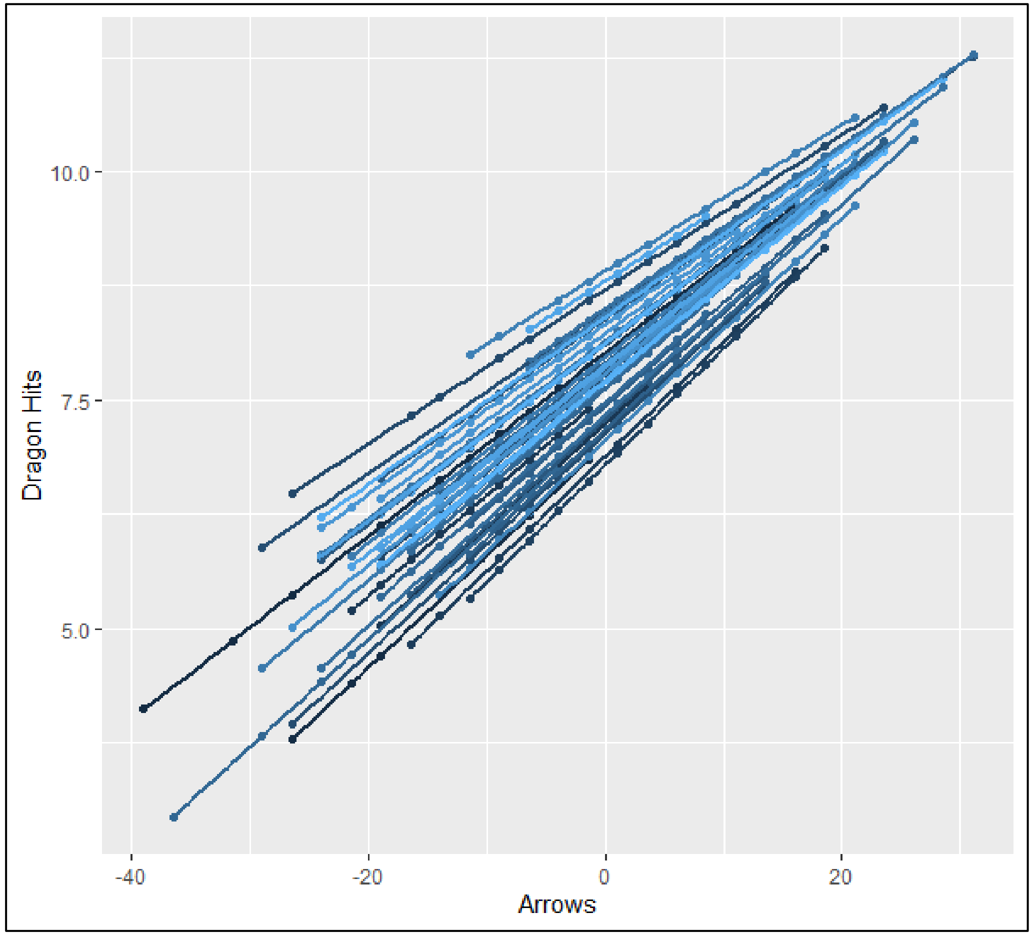
Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	7.8204	0.09882	45	79.14	<.0001
Arrows_C	0.1037	0.004712	45	22.00	<.0001

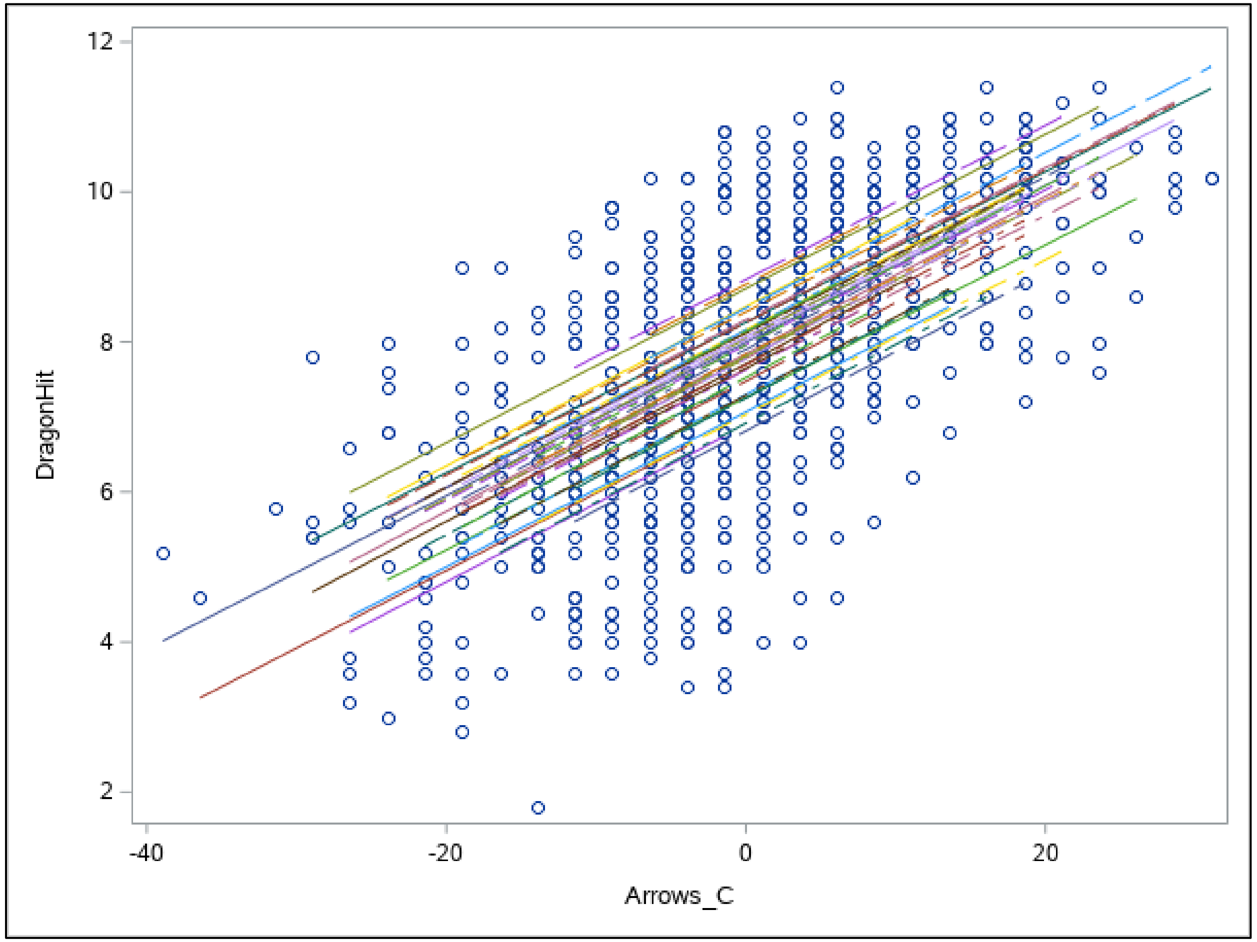
Type III Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
Arrows_C	1	45	484.14	<.0001

Example 3: Dragon Hits and Arrows

Model 3: Arrow Level-2 Random Intercepts and Slopes





Example 4: Dragon Gold and Training

Dataset Creation

- https://alexanderdemos.org/Mixed5.html [11]
- Got data and transformed it to fit the dragon theme

```
>setwd("C:/Users/Mark.Williamson.2/Desktop/Williamson Data/R/R_data/2022
Data")
>testds3 <-read.csv("Sim3level.csv")
>head(testds3)
  #Math<- Dragon_gold
  #ActiveTime <-TrainingTime
  #StudentID<- Knight
  #Classroom<- Commander
  #School <-Cathedral
>Dragon3 <-data.frame("DragonGold"=testds3$Math,
          "TrainingTime"=testds3$ActiveTime,
          "KnightID"=testds3$StudentID,
          "Commander"=testds3$Classroom, "Cathedral"=testds3$School)
```

	DragonGold	TrainingTime	KnightID	Commander	Cathedral	TrainingTime.C	Commander.2
1	55.4	0.7	1	1	C1	-4.140351	C1:1
2	54.3	0.8	2	1	C1	-4.040351	C1:1
3	61.4	1.3	3	1	C1	-3.540351	C1:1
4	56.1	7.5	4	1	C1	2.659649	C1:1
5	53.3	0.4	5	1	C1	-4.440351	C1:1
6	58.0	6.9	6	1	C1	2.059649	C1:1

Example 4: Dragon Gold and Training

Model 1: Training Standard Regression

R

```
>D4_M1 <- lm(DragonGold ~ TrainingTime.C,
        data=Dragon3)
>summary(D4_M1)


>ggplot(data=Dragon3, aes(x=TrainingTime.C,
        y=DragonGold)) +
        geom_point() +
        geom_smooth(method=lm, color="red")
```

SAS

```
PROC GLIMMIX data=Dragon3 method=MMPL;
        model DragonGold=TrainingTime_C/s;
        output out=D4M1_pred pred lcl ucl;
PROC SORT data=D4M1_pred;
        by TrainingTime_C;
PROC SGPLOT data=D4M1_pred noautolegend;
        band x=TrainingTime_C
                lower=lcl upper=ucl;
        scatter x=TrainingTime_C y=DragonGold;
        series x=TrainingTime_C y=pred;
```

Example 4: Dragon Gold and Training

Model 1: Training Standard Regression

```
Call:
lm(formula = DragonGold ~ TrainingTime.C, data = Dragon3)

Residuals:
    Min      1Q  Median      3Q     Max
-29.503  -7.641  -1.626   6.771  37.919

Coefficients:
               Estimate Std. Error t value Pr(>|t|)
(Intercept)     42.9605     0.4658  92.239   <2e-16 ***
TrainingTime.C   1.4648     0.1573   9.314   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 11.12 on 568 degrees of freedom
Multiple R-squared:  0.1325,	Adjusted R-squared:  0.131
F-statistic: 86.75 on 1 and 568 DF,  p-value: < 2.2e-16
```

Fit Statistics	
-2 Log Likelihood	4361.53
AIC (smaller is better)	4367.53
AICC (smaller is better)	4367.57
BIC (smaller is better)	4380.57
CAIC (smaller is better)	4383.57
HQIC (smaller is better)	4372.62
Pearson Chi-Square	70232.45
Pearson Chi-Square / DF	123.21

Parameter Estimates

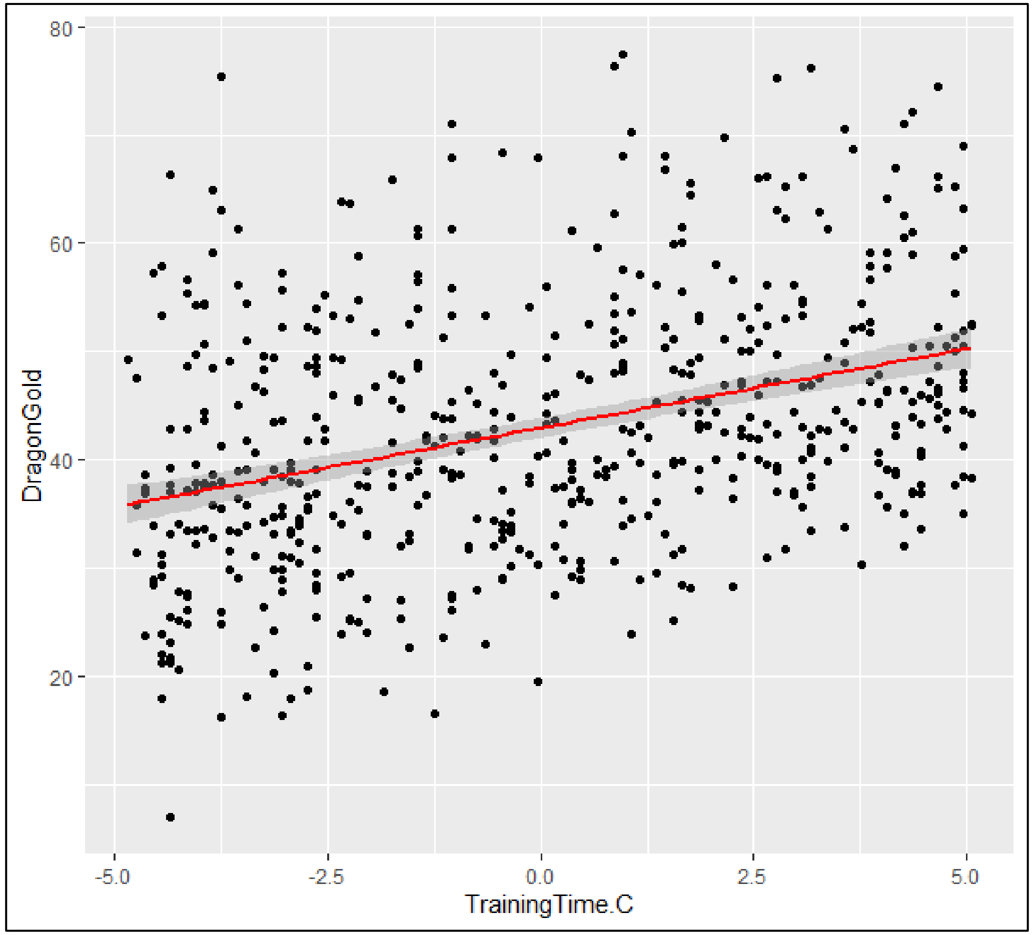
Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	42.9605	0.4649	568	92.40	<.0001
TrainingTime_C	1.4648	0.1570	568	9.33	<.0001
Scale	123.21	7.2986	.	.	.

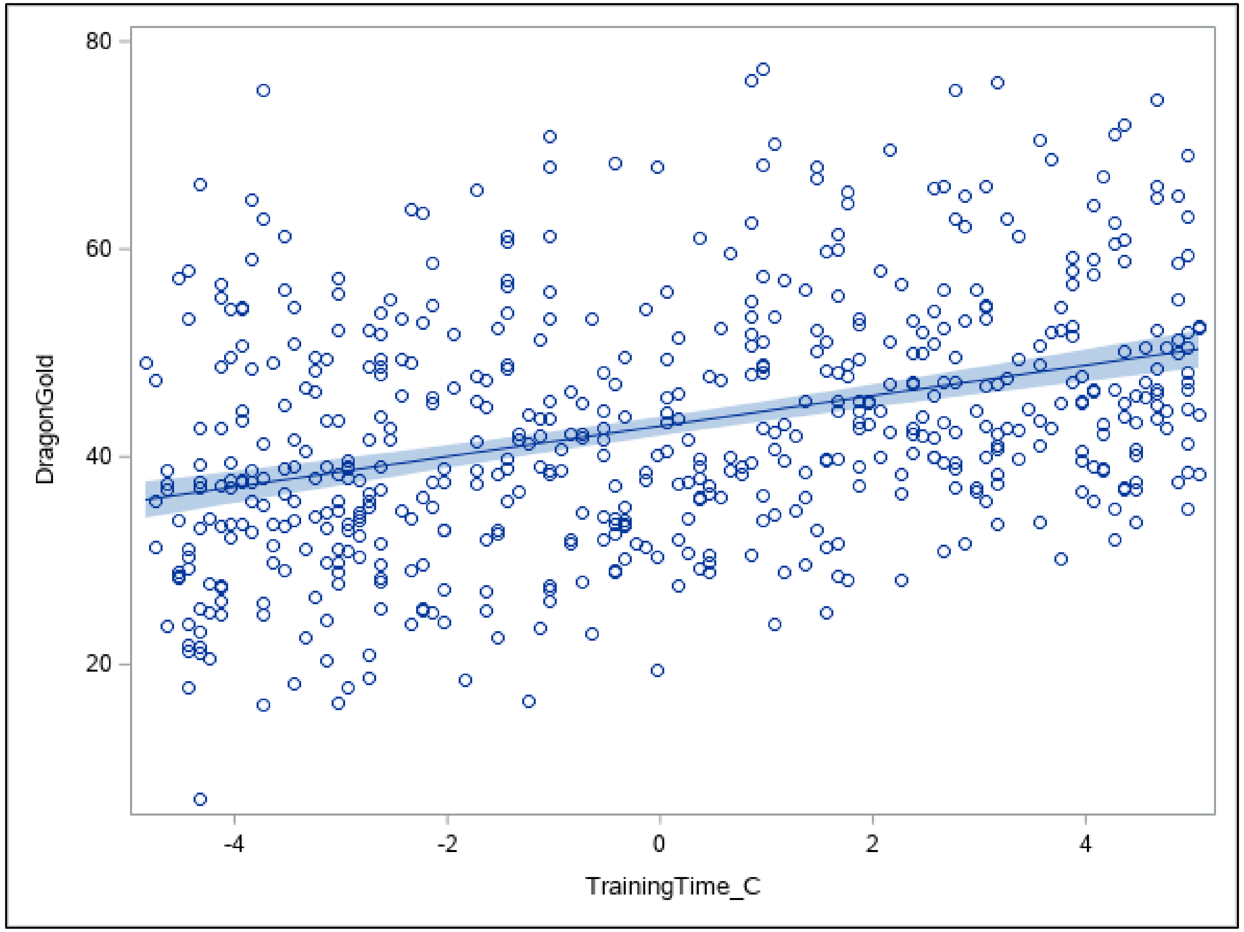
Type III Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
TrainingTime_C	1	568	87.06	<.0001

Example 4: Dragon Gold and Training

Model 1: Training Standard Regression





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Example 4: Dragon Gold and Training

Model 2: Training Level-2 Random Intercepts

R

```
>D4_M2 <- lmer(DragonGold ~ TrainingTime.C + (1|Commander.2), data=Dragon3, REML=F)
>summary(D4_M2)

>Dragon3$DragonGold_Pred <-predict(D4_M2, newdata=Dragon3)

>ggplot(data=Dragon3, aes(x=TrainingTime.C, y=DragonGold_Pred,
  group=Commander.2)) +
  geom_point(aes(color=Commander.2))+
  geom_smooth(method='lm', se=F, aes(colour=Commander.2))+
  xlab("Training Time") + ylab("Dragon Gold")+
  theme(legend.position = "none")
```

SAS

```
PROC GLIMMIX data=Dragon3 method=MMPL;
        class Commander_2;
        model DragonGold=TrainingTime_C/s;
        random Commander_2;
        output out=D4M2_pred pred lcl ucl;
PROC SORT data=D4M2_pred;
        by TrainingTime_C;
PROC SGPLOT data=D4M2_pred noautolegend;
        band x=TrainingTime_C lower=lcl
        upper=ucl/group=Commander_2
                transparency=.90;
        scatter x=TrainingTime_C y=DragonGold;
        series x=TrainingTime_C
        y=pred/group=Commander_2;
```

Example 4: Dragon Gold and Training

Model 2: Training Level-2 Random Intercepts

```
Linear mixed model fit by maximum likelihood  ['lmerMod']
Formula: DragonGold ~ TrainingTime.C + (1 | Commander.2)
   Data: Dragon3

     AIC      BIC   logLik deviance df.resid
  3401.4   3418.8  -1696.7   3393.4      566

Scaled residuals:
    Min      1Q  Median      3Q     Max
-3.3097 -0.6559 -0.0453  0.6428  3.1190

Random effects:
 Groups      Name        Variance Std.Dev.
 Commander.2 (Intercept) 109.16   10.448
 Residual                 17.54    4.189
Number of obs: 570, groups:  Commander.2, 30

Fixed effects:
               Estimate Std. Error t value
(Intercept)    43.99163    1.91588   22.96
TrainingTime.C  1.49248    0.06064   24.61

Correlation of Fixed Effects:
            (Intr)
TrainngTm.C 0.000
```

Fit Statistics	
-2 Log Likelihood	3393.45
AIC (smaller is better)	3401.45
AICC (smaller is better)	3401.52
BIC (smaller is better)	3407.05
CAIC (smaller is better)	3411.05
HQIC (smaller is better)	3403.24
Generalized Chi-Square	10000.17
Gener. Chi-Square / DF	17.54

Covariance Parameter Estimates

Cov Parm	Estimate	Standard Error
Commander_2	109.16	28.4361
Residual	17.5442	1.0677

Solutions for Fixed Effects

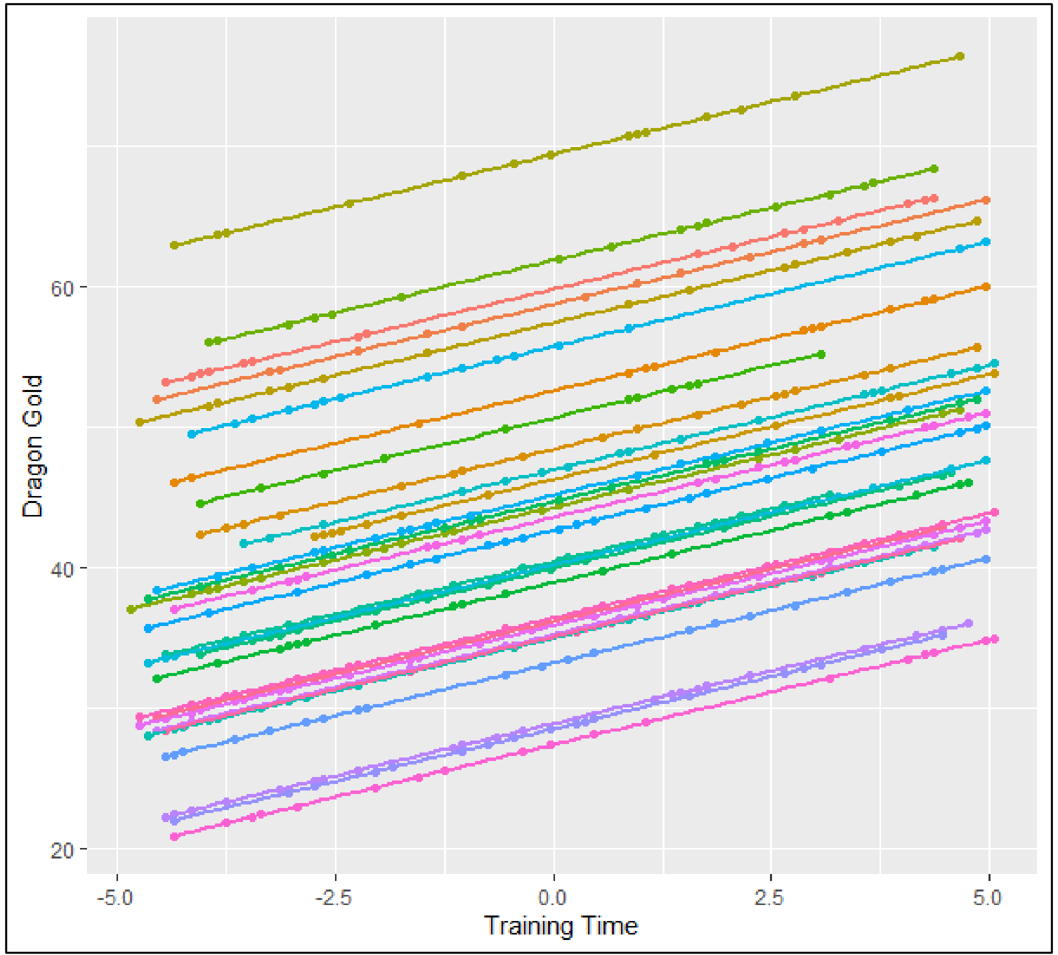
Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	43.9916	1.9159	29	22.96	<.0001
TrainingTime_C	1.4925	0.06064	539	24.61	<.0001

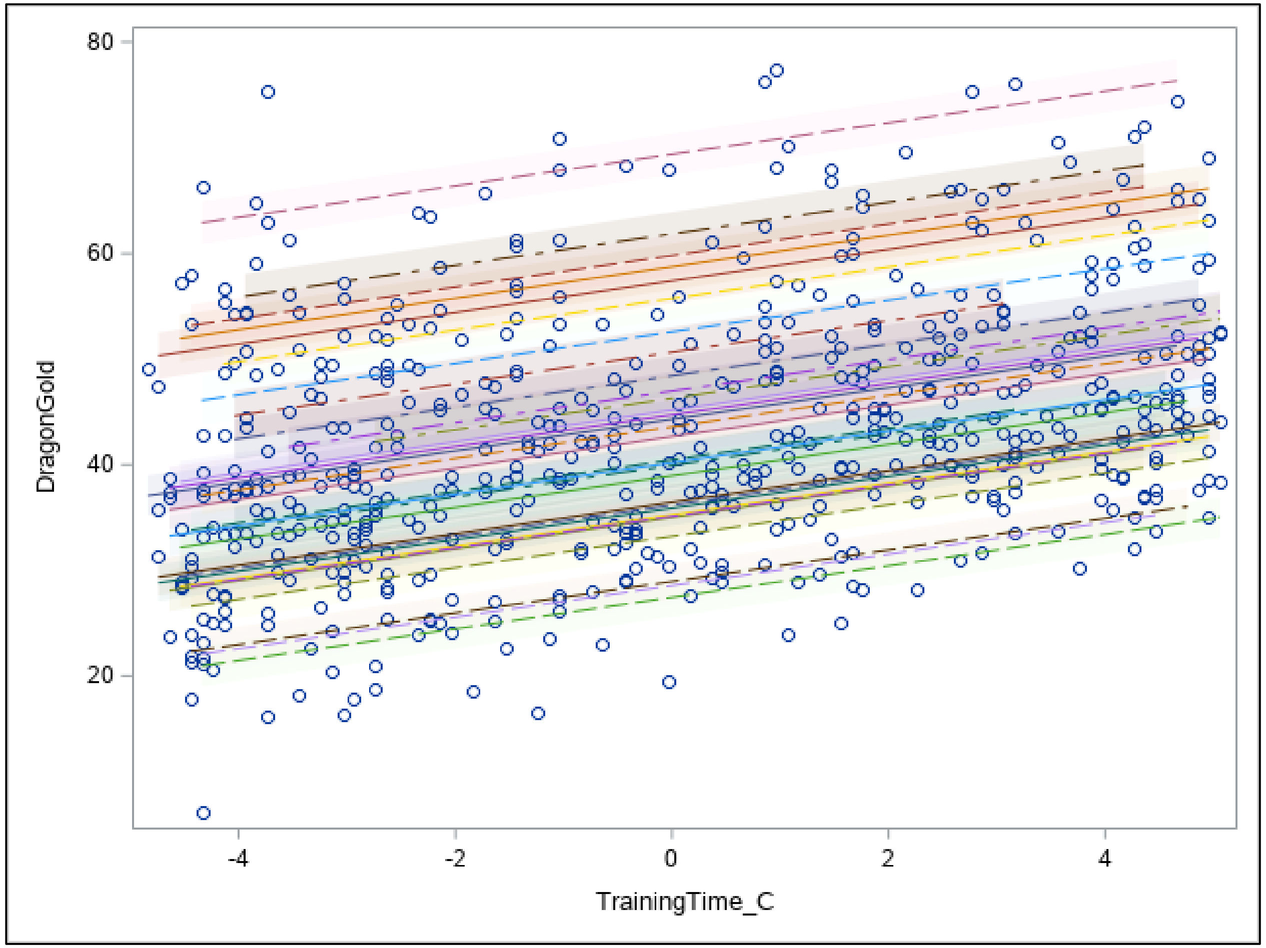
Type III Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
TrainingTime_C	1	539	605.84	<.0001

Example 4: Dragon Gold and Training

Model 2: Training Level-2 Random Intercepts





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Example 4: Dragon Gold and Training

Model 3: Training Level-3 Random Intercepts

R

```
>D4_M3.1 <- lmer(DragonGold ~ TrainingTime.C +
        (1|Cathedral) + (1|Commander.2),
        data=Dragon3, REML=F)

>summary(D4_M3.1)


>D4_M3.2 <- lmer(DragonGold ~ TrainingTime.C +
        (1|Cathedral) + (1|Cathedral:Commander),
        data=Dragon3, REML=F)

>summary(D4_M3.2)
```

SAS

```
PROC SGPLOT data=D4M3_pred noautolegend;
        where Cathedral='C2';
        band x=TrainingTime_C lower=lcl
        upper=ucl/group=Commander_2 transparency=.90;
        scatter x=TrainingTime_C y=DragonGold;
        series x=TrainingTime_C y=pred/group=Commander_2;
        yaxis ranges=(0-80); title "C2";
PROC SGPLOT data=D4M3_pred noautolegend;
        where Cathedral='C3'; title "C2";
        band x=TrainingTime_C lower=lcl
        upper=ucl/group=Commander_2 transparency=.90;
        scatter x=TrainingTime_C y=DragonGold;
        series x=TrainingTime_C y=pred/group=Commander_2;
        yaxis ranges=(0-80); title "C3";
        yaxis ranges=(0-80); title "C1";
```

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Example 4: Dragon Gold and Training

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Model 3: Training Level-3 Random Intercepts

```
Linear mixed model fit by maximum likelihood  ['lmerMod']
Formula: DragonGold ~ TrainingTime.C + (1 | Cathedral) + (1 | Cathedral:Commander)
   Data: Dragon3

     AIC      BIC   logLik deviance df.resid
  3384.9   3406.7  -1687.5   3374.9      565

Scaled residuals:
    Min      1Q  Median      3Q     Max
-3.3181 -0.6554 -0.0496  0.6355  3.0840

Random effects:
 Groups              Name        Variance Std.Dev.
 Cathedral:Commander (Intercept) 44.67    6.684
 Cathedral           (Intercept) 60.43    7.774
 Residual                        17.54    4.189
Number of obs: 570, groups:  Cathedral:Commander, 30; Cathedral, 3

Fixed effects:
               Estimate Std. Error t value
(Intercept)    44.44829    4.65891    9.54
TrainingTime.C  1.49291    0.06062   24.63

Correlation of Fixed Effects:
           (Intr)
TrainngTm.C 0.000
```

Fit Statistics	
-2 Log Likelihood	3393.45
AIC (smaller is better)	3401.45
AICC (smaller is better)	3401.52
BIC (smaller is better)	3407.05
CAIC (smaller is better)	3411.05
HQIC (smaller is better)	3403.24
Generalized Chi-Square	10000.17
Gener. Chi-Square / DF	17.54

Covariance Parameter Estimates

Cov Parm	Estimate	Standard Error
Commander(Cathedral)	109.16	28.4361
Residual	17.5442	1.0677

Solutions for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	43.9916	1.9159	29	22.96	<.0001
TrainingTime_C	1.4925	0.06064	539	24.61	<.0001

Type III Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
TrainingTime_C	1	539	605.84	<.0001

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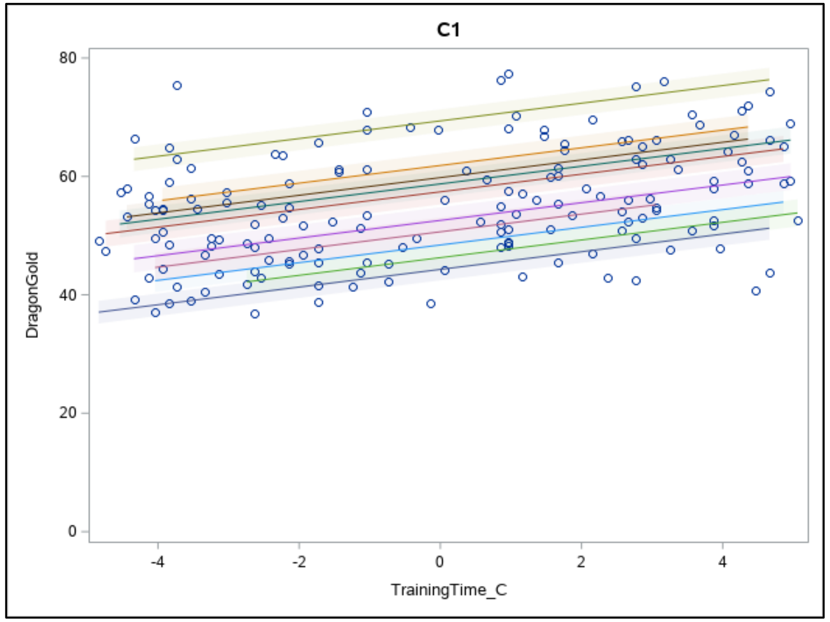
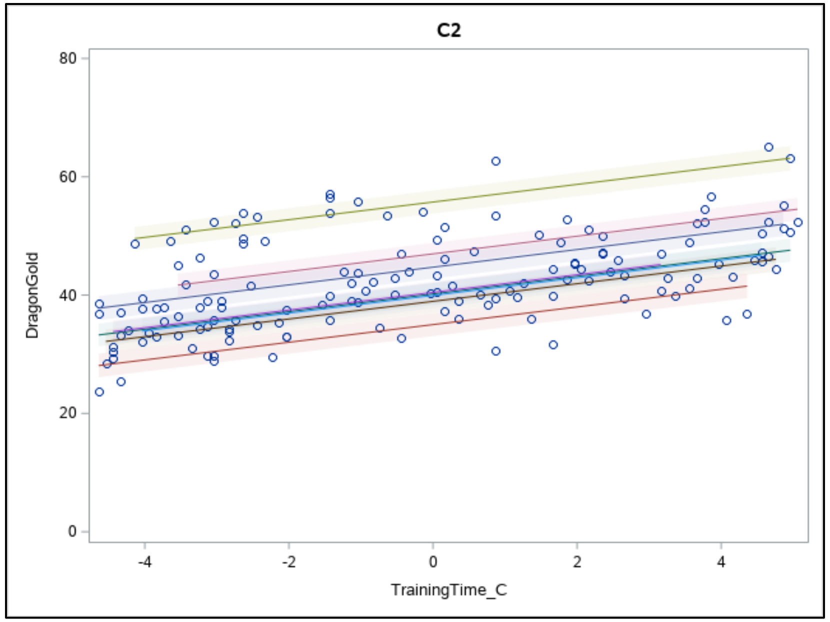
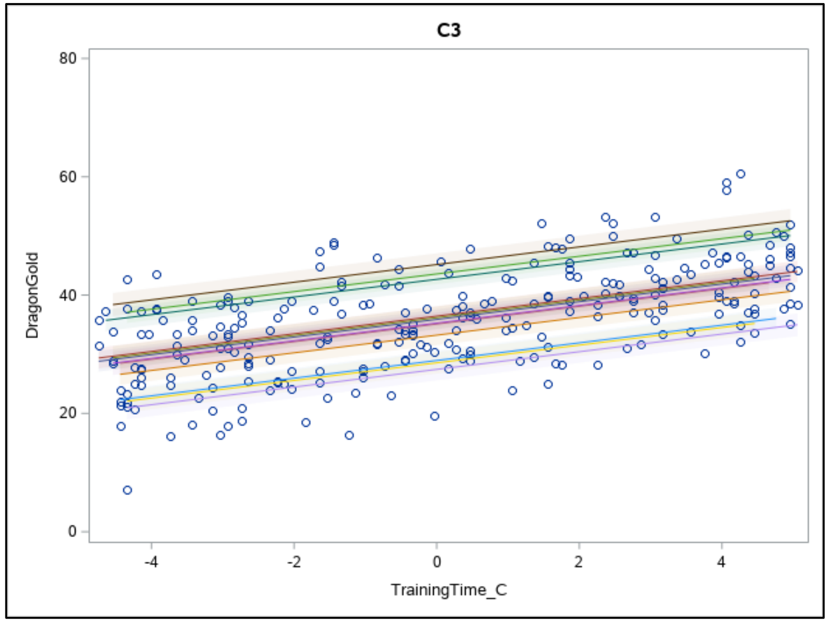
Example 4: Dragon Gold and Training

Model 3: Training Level-2 Random Intercepts and Slopes







Example 4: Dragon Gold and Training

Model 4: Training Level-3 Random Intercepts and Slopes

R

```
>D4_M4 <- lmer(DragonGold ~ TrainingTime.C + (1+TrainingTime.C||Cathedral) +
          (1+TrainingTime.C|Cathedral:Commander), data=Dragon3, REML=F)
>summary(D4_M4)
>Dragon3$DragonGold_Pred2 <-predict(D4_M4, newdata=Dragon3)

>ggplot(data=Dragon3, aes(x=TrainingTime.C, y=DragonGold_Pred2,
 group=Commander)) +
 facet_grid(~Cathedral) + geom_point(aes(color=Commander))+
 geom_smooth(method='lm', se=F, aes(colour=Commander))+
 xlab("Training Time") + ylab("Dragon Gold")+
 theme(legend.position = "none")
```

SAS

```
PROC SGPLOT data=D4M4_pred noautolegend;
        where Cathedral='C2';
        band x=TrainingTime_C lower=lcl
        upper=ucl/group=Commander transparency=.90;
        scatter x=TrainingTime_C y=DragonGold;
        series x=TrainingTime_C y=pred/group=Commander;
        yaxis ranges=(0-80); title "C2";
PROC SGPLOT data=D4M4_pred noautolegend;
        where Cathedral='C3';
        band x=TrainingTime_C lower=lcl
        upper=ucl/group=Commander transparency=.90;
        scatter x=TrainingTime_C y=DragonGold;
        series x=TrainingTime_C y=pred/group=Commander;
        yaxis ranges=(0-80); title "C3";
```

Example 4: Dragon Gold and Training

Model 4: Training Level-3 Random Intercepts and Slopes

```
Linear mixed model fit by maximum likelihood  ['lmerMod']
Formula: DragonGold ~ TrainingTime.C + ((1 | Cathedral) + (0 + TrainingTime.C |
Cathedral)) + (1 + TrainingTime.C | Cathedral:Commander)
   Data: Dragon3

     AIC      BIC   logLik deviance df.resid
  3353.7   3388.5  -1668.9   3337.7      562

Scaled residuals:
     Min       1Q   Median       3Q      Max
-3.03150 -0.64800 -0.02983  0.65167  2.97169

Random effects:
 Groups              Name           Variance Std.Dev. Corr
 Cathedral.Commander (Intercept)    46.05782 6.7866
                     TrainingTime.C  0.17298 0.4159   0.14
 Cathedral           TrainingTime.C  0.06047 0.2459
 Cathedral.1         (Intercept)    62.33496 7.8952
 Residual                           15.36809 3.9202
Number of obs: 570, groups:  Cathedral:Commander, 30; Cathedral, 3

Fixed effects:
               Estimate Std. Error t value
(Intercept)     44.3677     4.7311   9.378
TrainingTime.C   1.4418     0.1719   8.385

Correlation of Fixed Effects:
            (Intr)
TraingTm.C 0.016
```

Fit Statistics	
-2 Log Likelihood	3338.03
AIC (smaller is better)	3352.03
AICC (smaller is better)	3352.22
BIC (smaller is better)	3345.72
CAIC (smaller is better)	3352.72
HQIC (smaller is better)	3339.34
Generalized Chi-Square	8763.03
Gener. Chi-Square / DF	15.37

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error
Intercept	Commander(Cathedral)	46.1129	12.7761
TrainingTime_C	Commander(Cathedral)	0.1722	0.07246
Intercept	Cathedral	60.5906	53.0752
TrainingTime_C	Cathedral	0.05829	0.06830
Residual		15.3737	0.9618

Solutions for Fixed Effects

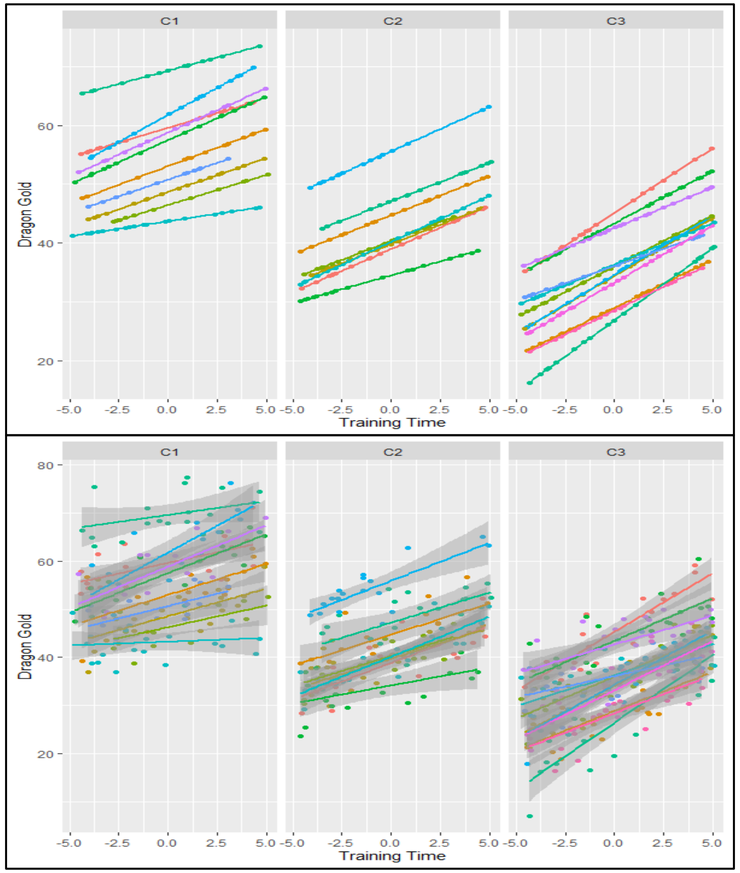
Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	44.3533	4.6694	0	9.50	.
TrainingTime_C	1.4425	0.1697	0	8.50	.

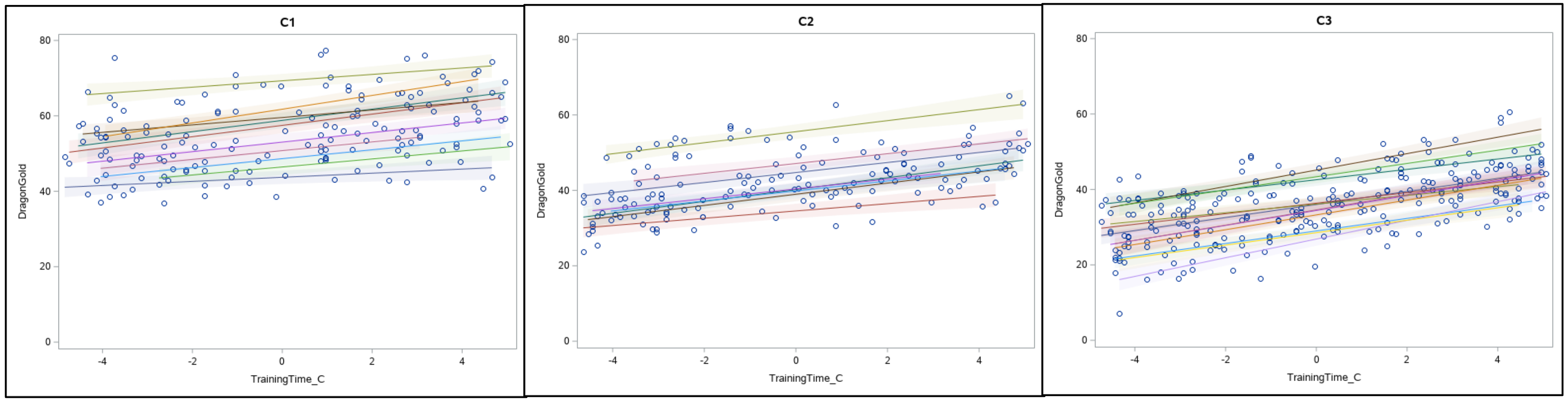
Type III Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
TrainingTime_C	1	0	72.22	.

Example 4: Dragon Gold and Training

Model 4: Training Level-3 Random Intercepts and Slopes





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Conclusions

- Multilevel modeling deals with data that have non-independent grouping (aka, multi-levels, nesting, hierarchies, etc..)
- Not all regressions require multilevel modeling
- Always include fixed and random effects
- Intercepts and slopes can be made random
- With careful planning and attention, sophisticated results are possible

Please take the post-test and survey:

Post-test: https://und.qualtrics.com/jfe/form/SV_er1nW9wUcC9xrZI
Survey: https://und.qualtrics.com/jfe/form/SV_1FwNCWiobpKVadw

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