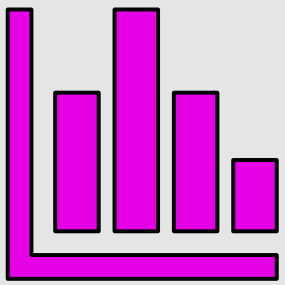
The Wide World of Distributions

BERDC Special Topics Talk 7



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DAKOTA CANCER COLLABORATIVE ON TRANSLATIONAL ACTIVITY

Dr. Mark Williamson

Biostatistics, Epidemiology, and Research Design Core

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Opening

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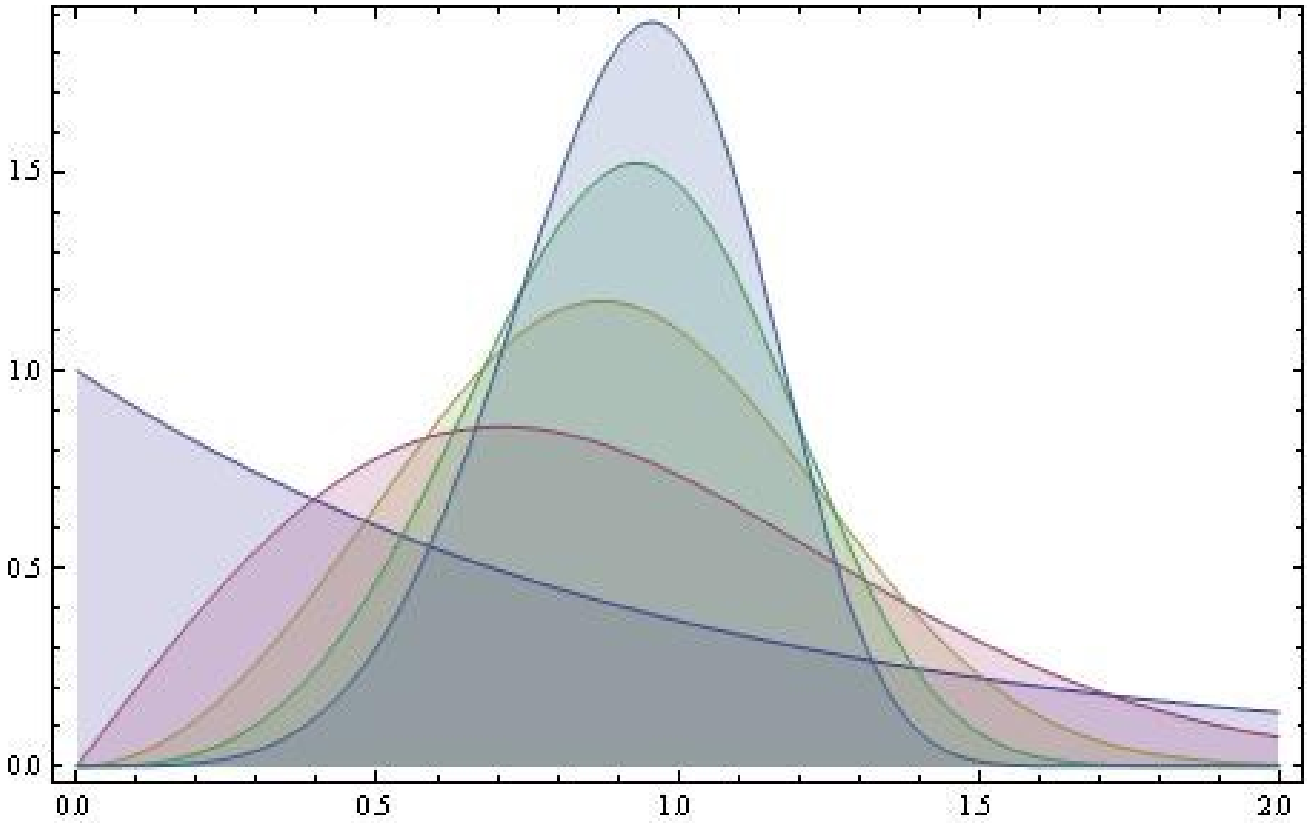
Goal: learn about statistical distributions

- Better statistical understanding
- Better statistical modeling
- Better statistical insights



Before Moving On:

Pre-test: https://und.qualtrics.com/jfe/form/SV_1QRRQKFLG086iuW

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Definition

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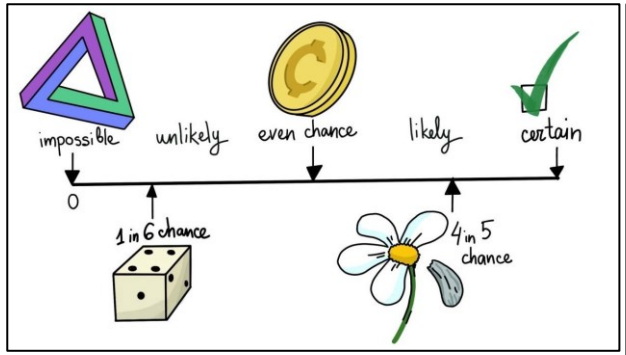
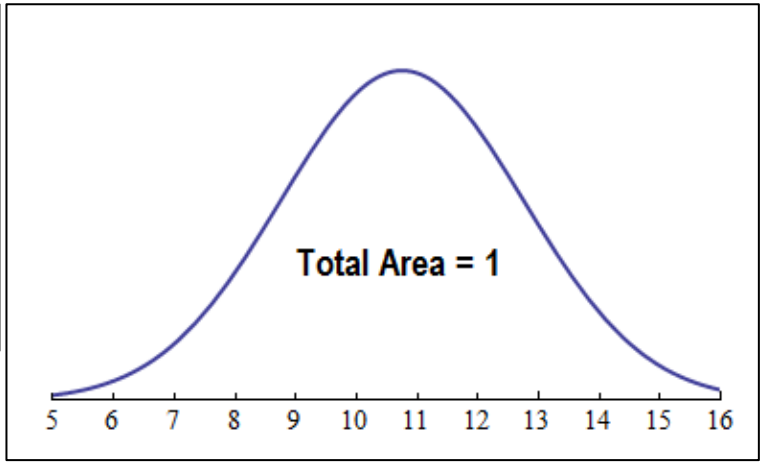
"Function that shows the possible values for a variable and how often they occur"

Properties

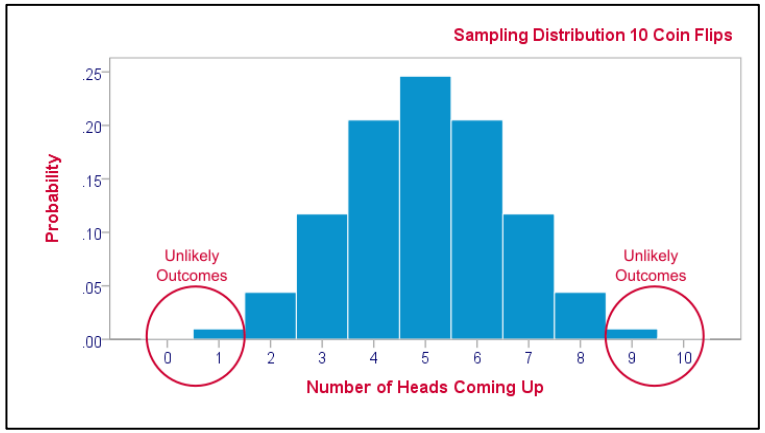
- probability that x can take a specific value is p(x)
- p(x) is non-negative for all real x
- sum of p(x) over all x equals 1





From the variable's perspective

- variables are sampled from random distributions that occur naturally
- each random distribution tells a different story about the processes that produce that variable
- each distribution has a mean, variance, and probability density function



"Mathematical construct to describe how variables are generated"

"Mathematical description of how data conceivably can be produced"

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Variable origins

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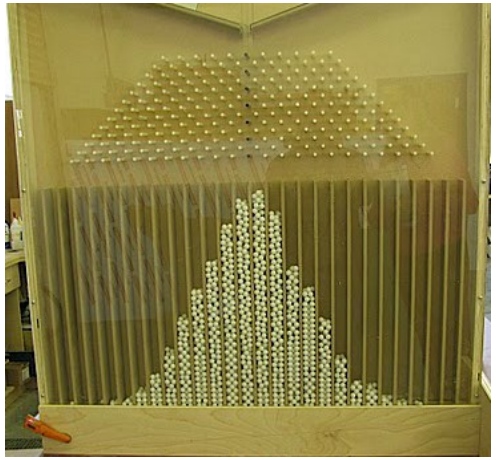
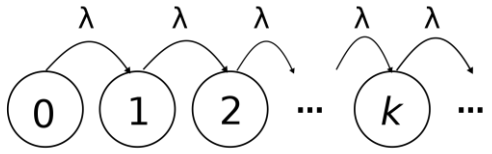
Random generation processes

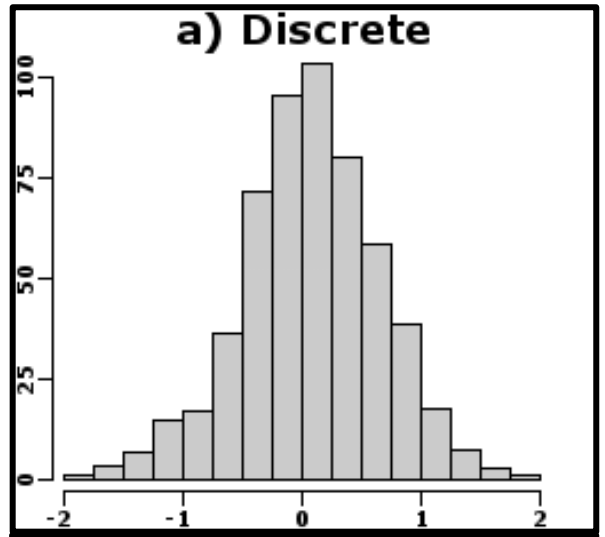
- Gaussian – random dispersal from central point; random diffusion from mean
- Bernoulli Trial – discrete event with only two outcomes (success/failure) with constant probability of success: P(success)= *p*.
- Poisson point process – events that occur individually in continuous time (or space)

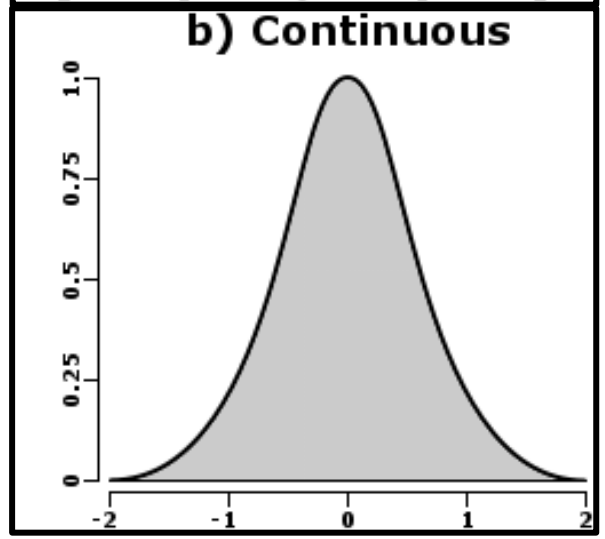
Broad classification

- Discrete - takes on only integer or 'count' variables
- Continuous – takes on any numerical value within range









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Properties

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Name	Notation	Range	Mean(x)	Var(x)	Explanation
Normal	$N(\mu,\sigma^2)$	$-\infty < x < \infty$	μ	σ^2	x=dispersal from a central point, or a diffusion through a Gaussian filter, with variance independent of mean.
Log-Normal	$Lognormal(\mu,\sigma^2)$	$x > 0$	$\exp(\mu+\sigma^2/2)$	$[\exp(\sigma^2)-1]^* \exp(2\mu+\sigma^2)$	x =probability distribution whose logarithm is normally distributed.
Exponential	$exp(\beta)$	$x > 0$	β	β^2	x =time between events that occur at a rate of $\lambda = 1/\beta$.
Gamma	$Gamma(k,\theta)$	$x > 0$	$k\theta$	$k\theta^2$	x =time it takes for *k* events to occur within a rate of $\lambda = 1/\theta$, or the sum of *k* exponential events.
Beta	$Beta(a,b)$	$0 < x < 1$	$\frac{a}{a+b}$	$\frac{ab}{(a+b)^2+(a+b+1)}$	x =distribution of probabilities based on *a* successes and *b* failures, where both *a* and *b* >1.
Binomial	$Bin(n,p)$	x = 0, 1, 2…	np	$np(1-p)$	x =number of positive events out of n trials each with a probability of success *p*.
Geometric	$G(p)$	x = 1, 2, 3…	$\frac{1}{p}$	$\frac{1-p}{p^2}$	x =number of trials, with probability of success *p*, that are needed to obtain one success.
Negative Binomial	$NB(n,p)$	x = 0, 1, 2…	$\frac{k(1-p)}{p}$	$\frac{k(1-p)}{p^2}$	x =number of failures before *k* successes occur in sequential independent trials, all with the same probability of success, *p*.
Poisson	$Poisson(\lambda)$	x = 0, 1, 2…	λ	λ	x =count of items in a standardized unit of effort that occur at rate λ.

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Statistical tests

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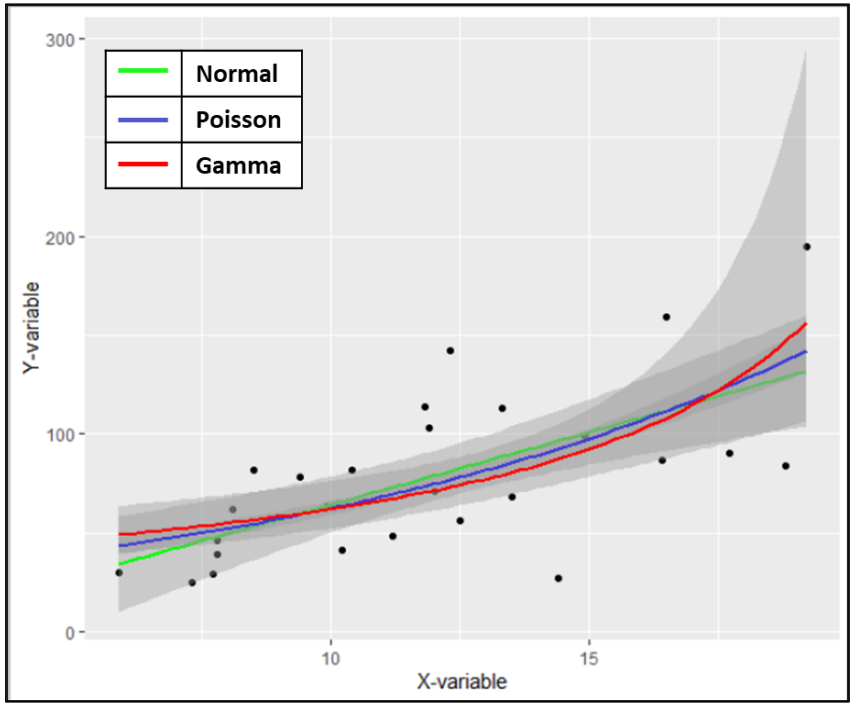
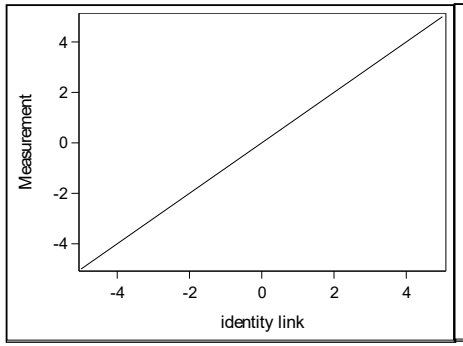
What sort of tests use distributions?

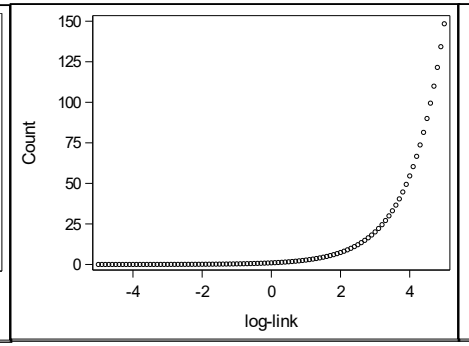
- Parametric tests assume a normal (Gaussian) distribution
  - ANOVA, t-test, linear regression, etc.
- Generalized linear models can use various distributions
  - 'generalized' refers to models that don't assume a normal distribution
  - Common Examples are Poisson and Logistic regression

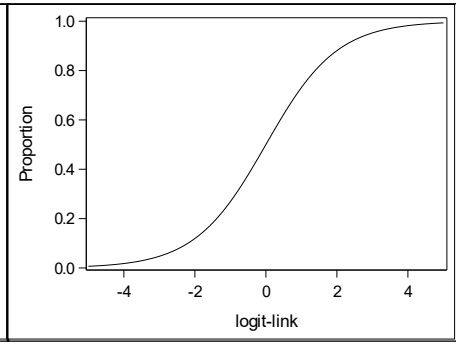


How to plot results?

- Each distribution has a standard link function
- Common ones are 'identity', 'log' and 'logit'







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Taxonomy

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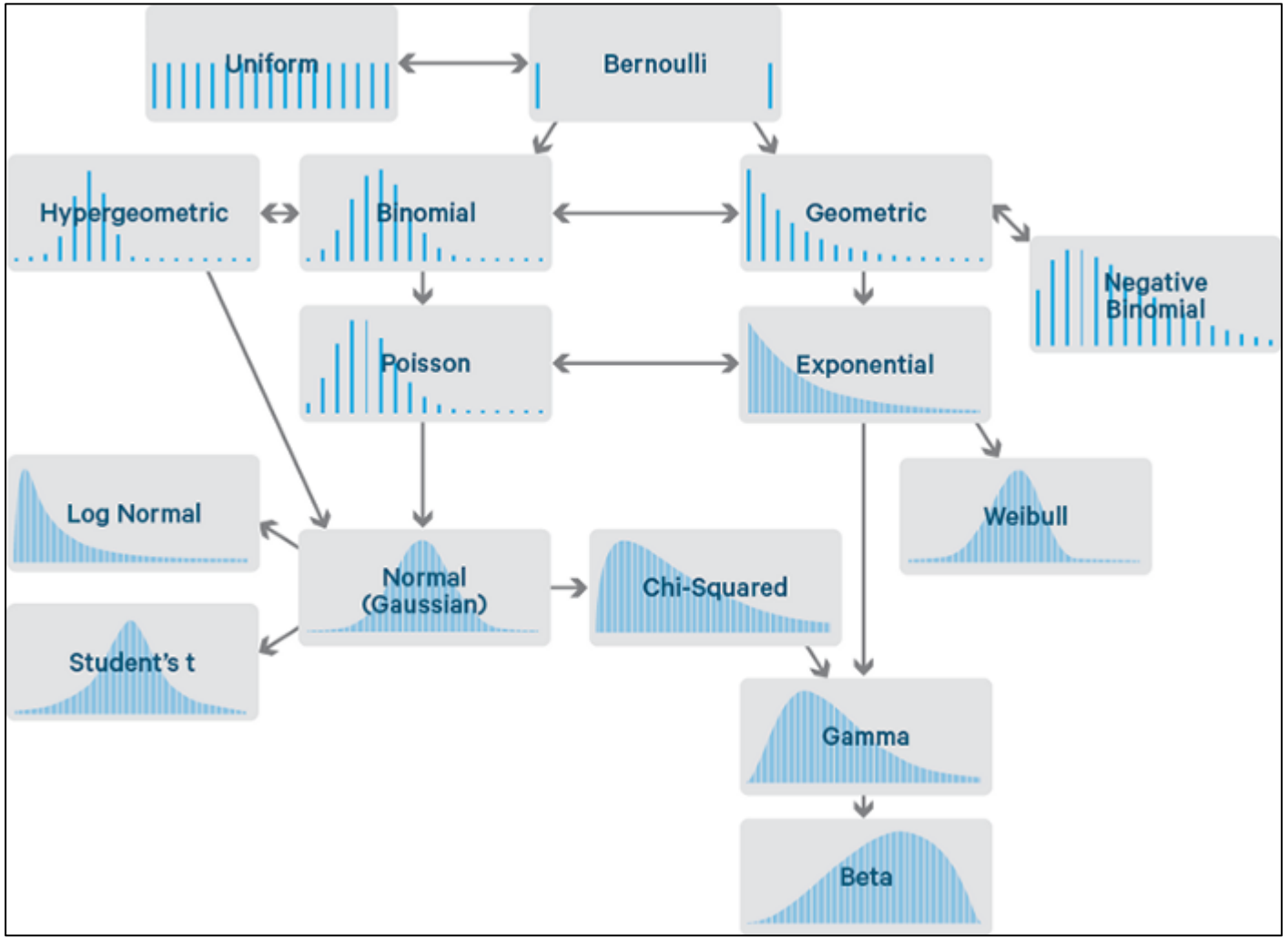
Taxonomy of Random Variables

Below is a taxonomy of common distributions, classified primarily by the topology of their support.

- finite
  - Dirac
  - Bernoulli
- countable
  - binomial
  - geometric
  - Poisson
- interval
  - uniform
  - beta
- half-line
  - exponential
  - gamma
- unbounded
  - normal
  - Cauchy
  - (Lévy) stable
- *transforms*
  - (generalized) Pareto
  - inverse gamma
  - lognormal
- *mixtures*
  - (Gosset) Student t
  - negative-binomial
- *non-parametric*
  - empirical



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Taxonomy cont.

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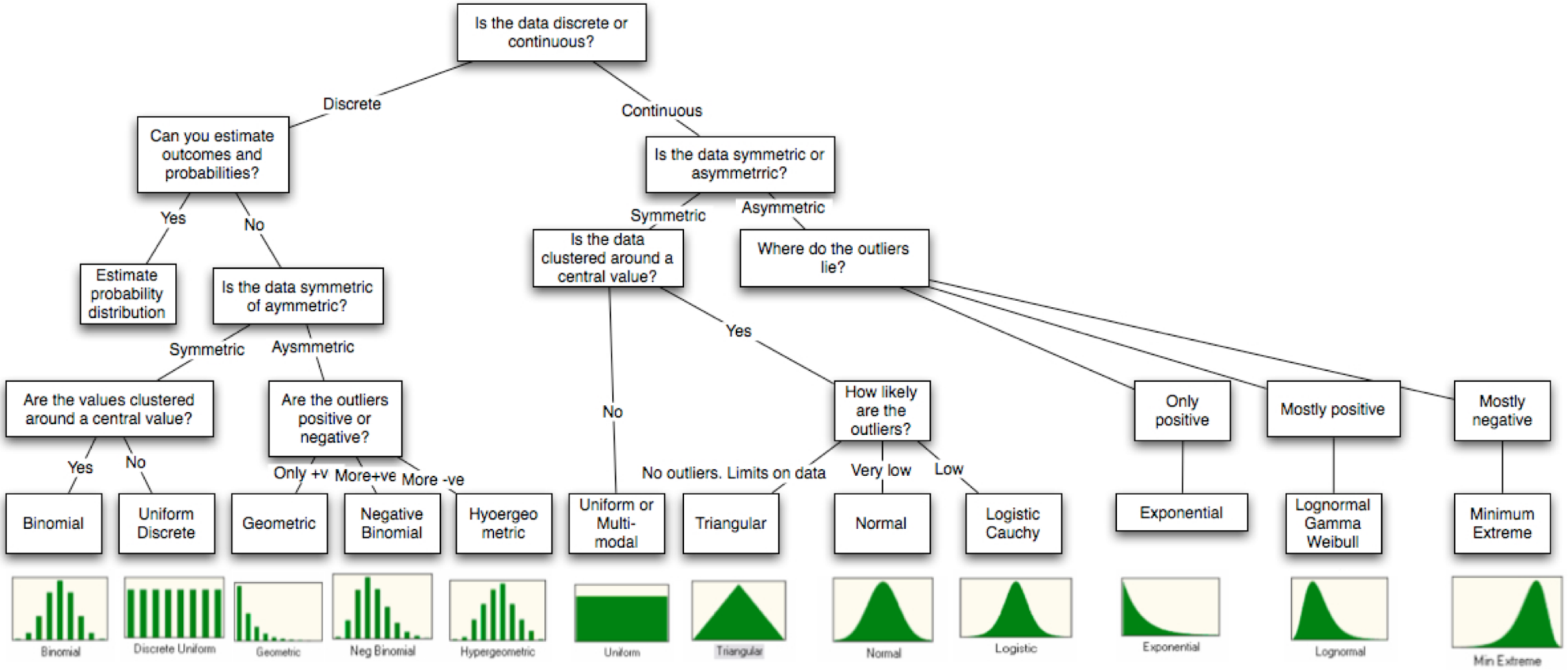

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Normal & Log Normal

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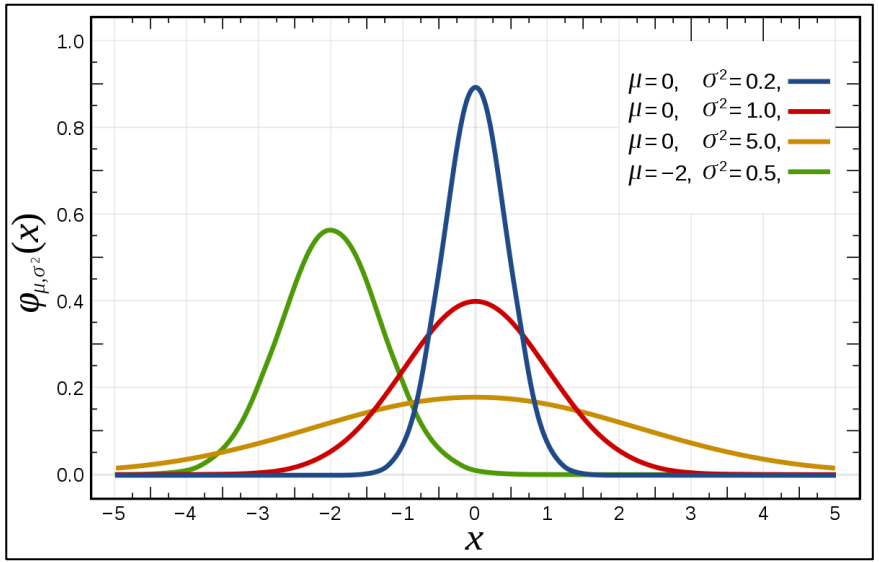
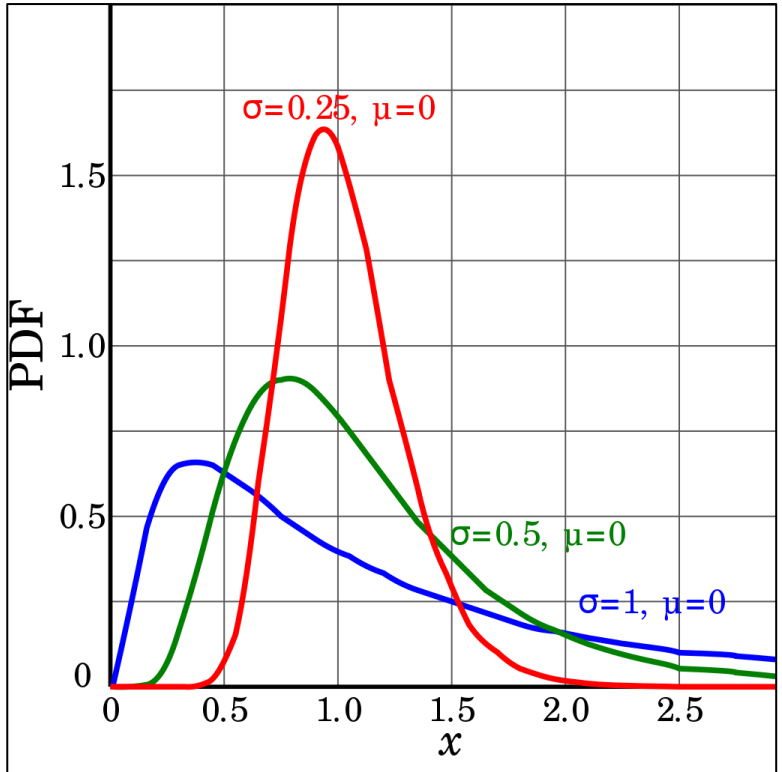
Normal

- Distribution models dispersion from central point
- Continuous, unbounded, and with a variance independent of its mean



Log Normal

- Distribution whose log-transformation is normal



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Binary & Binomial

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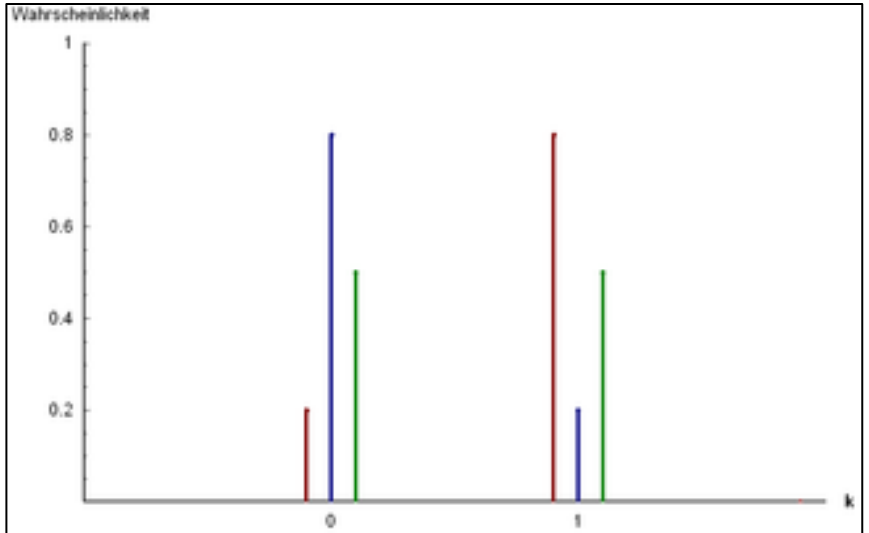
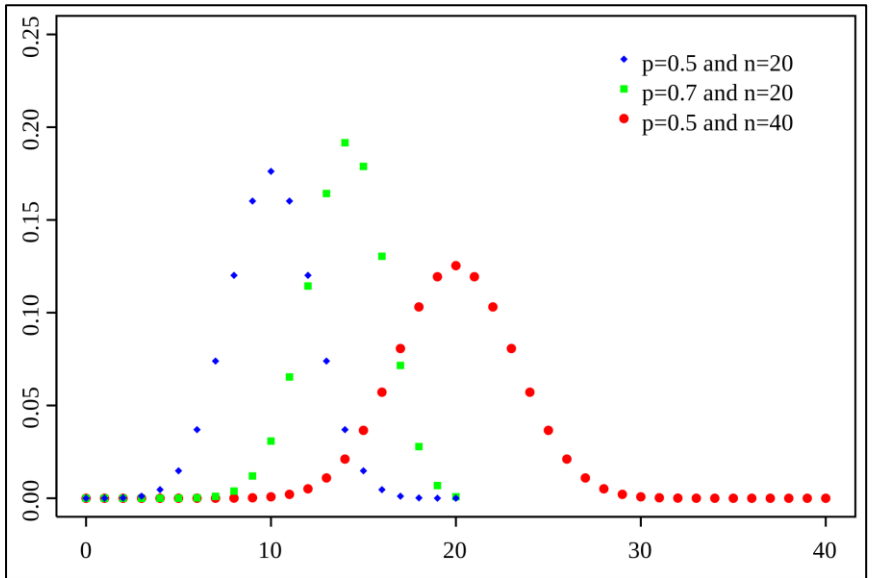
Binary

- Distribution that models binary (two) outcomes
- Aka ‘Bernoulli trials’
- Example: coin flips



Binomial

- Distribution that models the number of success from Bernoulli trials
- Example: coin flips



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Geometric, Poisson, & Negative Binomial

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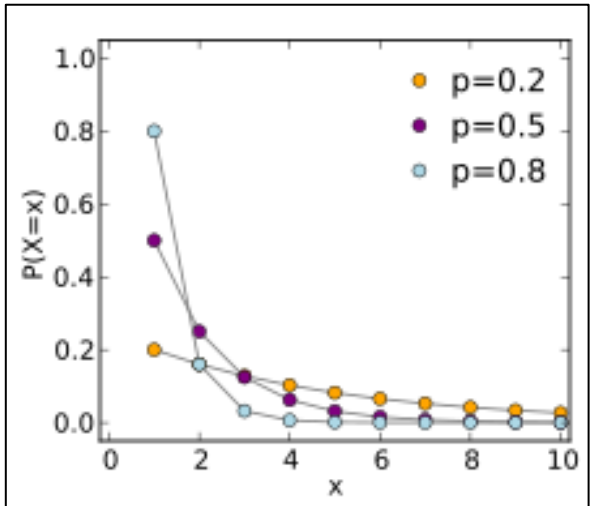
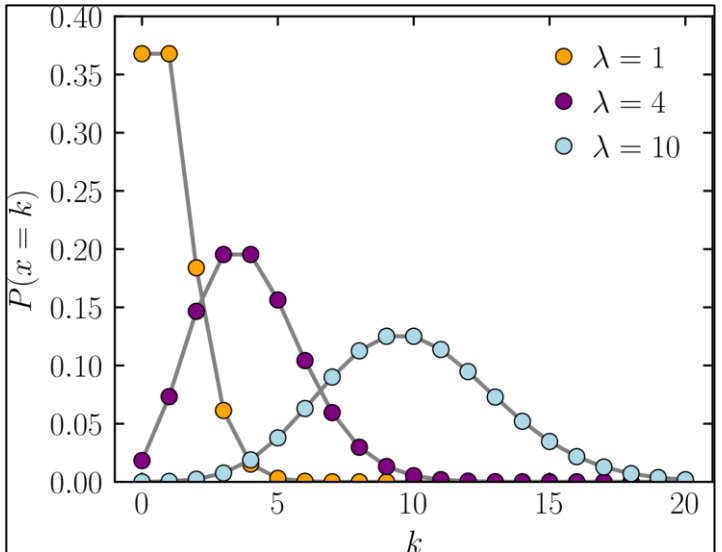
Geometric

- Distribution that models number of Bernoulli trials needed for one success
- Start at 0 or 1
- Example: coin flips until heads



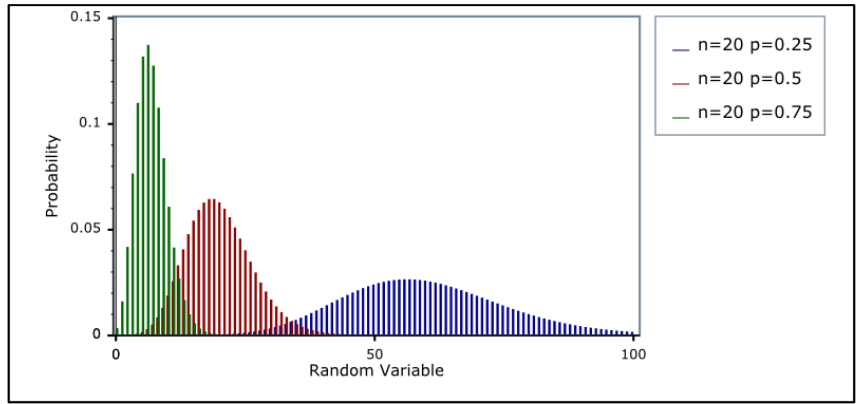
Poisson

- Distribution that models the number of counts occurring at a certain rate
- Example: number of patrons entering restaurant an hour, or number of trees per acre of woods



Negative Binomial

- Distribution that models the number of failures needed for a certain number of successes
- Can also be used for 'count' data as a sort of over-dispersed correction for Poisson
- Example: flip coins until 3 heads, count # of tails



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Exponential, Gamma, & Beta

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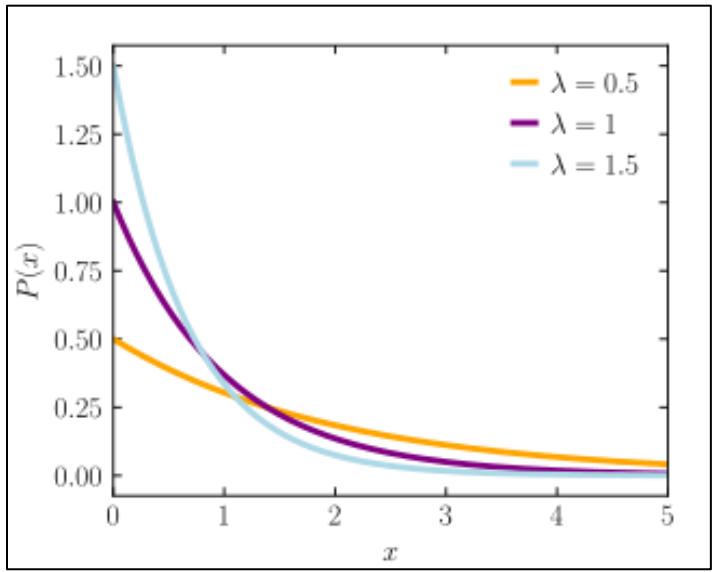
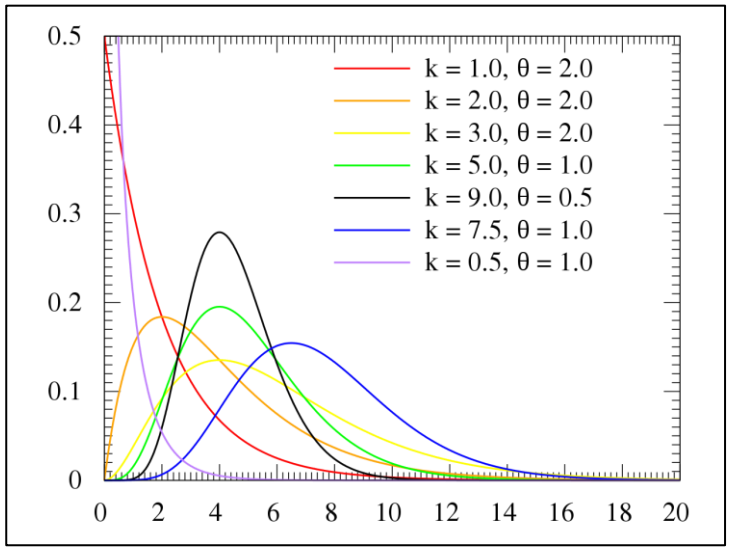
Exponential

- Distribution that models the inter-arrival time between Poisson process events
- Example: time between arrive of patrons into a restaurant



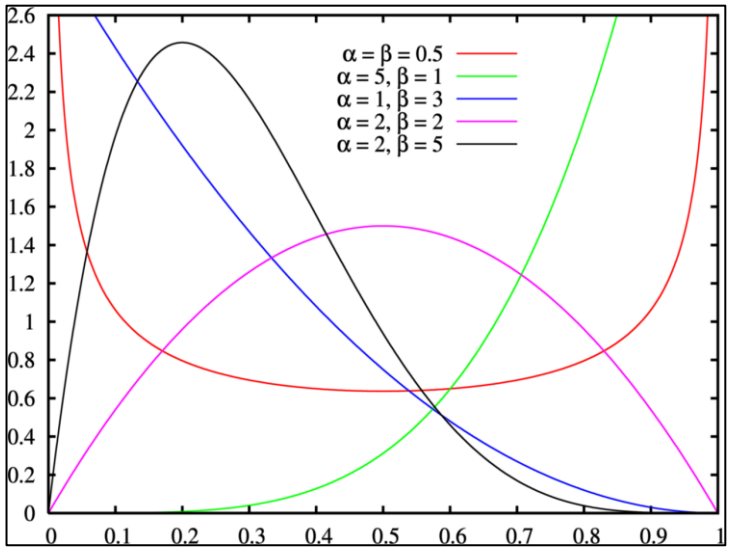
Gamma

- Distribution that models the total arrival time for a number of Poisson process events
- Example: time it takes for 5 patrons to enter a restaurant



Beta

- Distribution that models a proportion of success between 0 and 1
- Example: proportion of hits out of at-bats



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Other distributions

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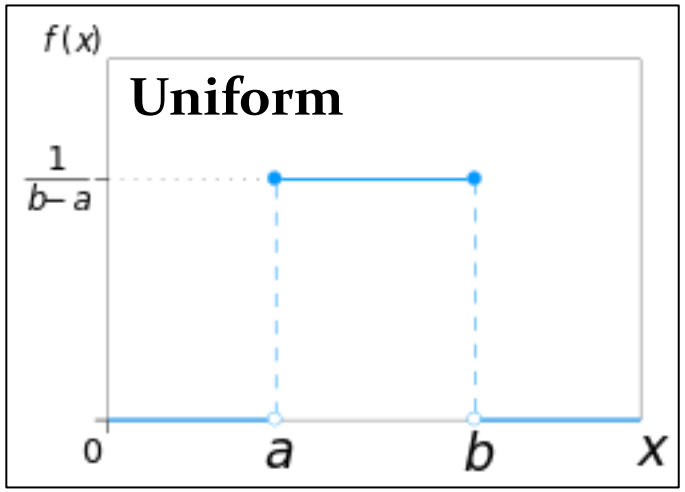

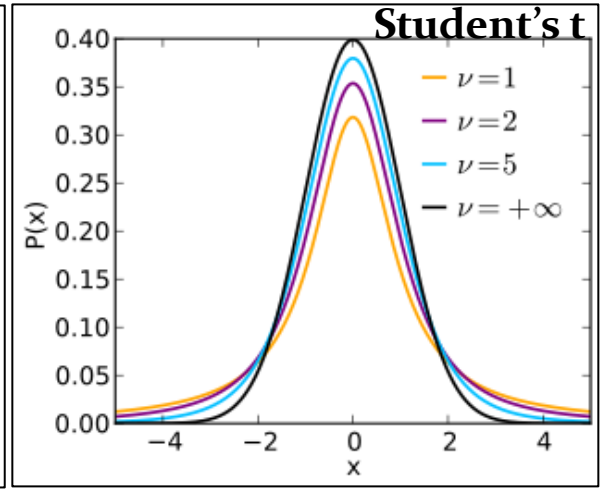

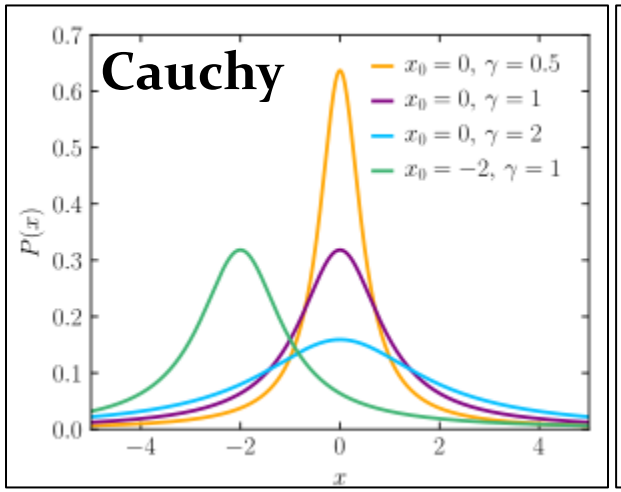

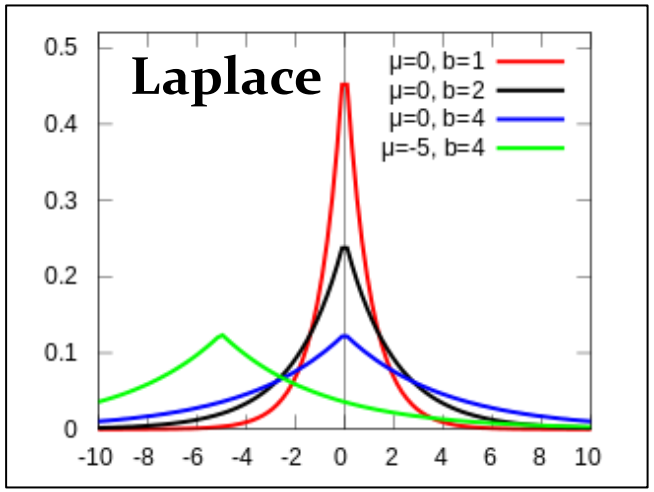

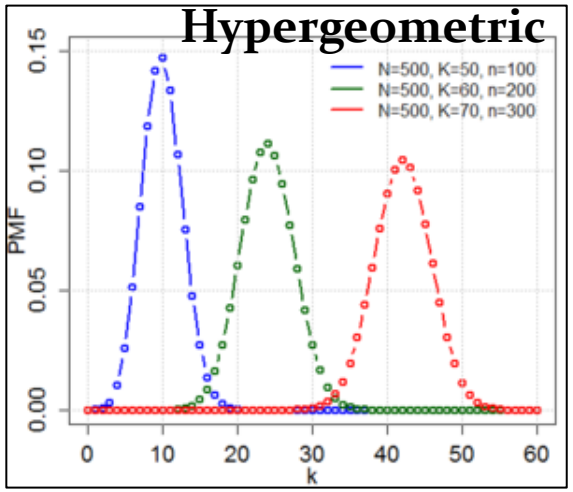

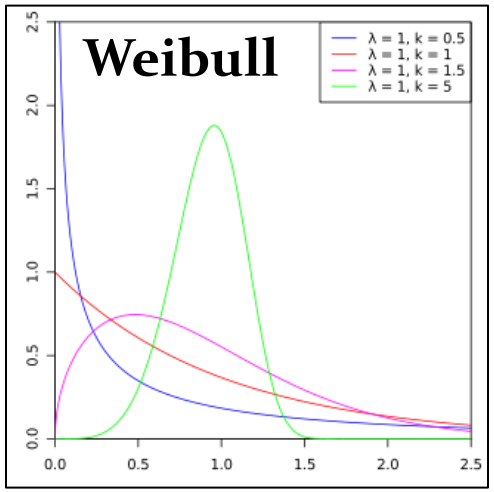

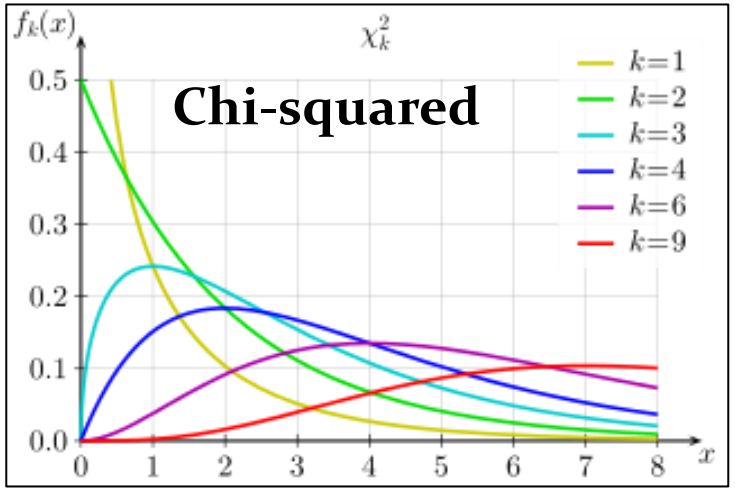

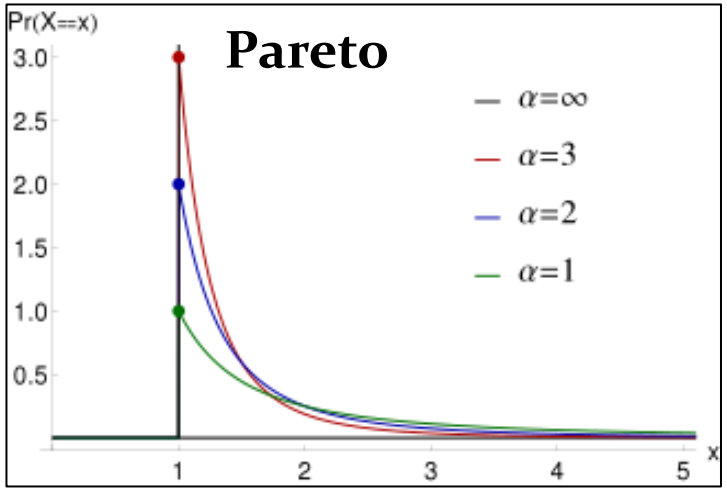

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SAS examples

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DIST=	Distribution	Default Link Function	Numeric Value
BETA	beta	logit	12
BINARY	binary	logit	4
BINOMIAL | BIN | B	binomial	logit	3
EXPONENTIAL | EXPO	exponential	log	9
GAMMA | GAM	gamma	log	5
GAUSSIAN | G | NORMAL | N	normal	identity	1
GEOMETRIC | GEOM	geometric	log	8
INVGAUSS | IGAUSSIAN | IG	inverse Gaussian	inverse squared	6
		(power(–2))	
LOGNORMAL | LOGN	lognormal	identity	11
MULTINOMIAL | MULTI | MULT	multinomial	cumulative logit	NA
NEGBINOMIAL | NEGBIN | NB	negative binomial	log	7
POISSON | POI | P	Poisson	log	2
TCENTRAL | TDIST | T	t	identity	10
BYOBS(*variable*)	multivariate	varied	NA

SAS Programmer | Sign Out

Distributions Examples.sas

CODE | LOG | RESULTS

```
*Create and view distributions;
DATA rand;
    do i=1 to 10000;
        * generate random values for 10000 observations;
        trt="trt";
        mynormal=rand("NORMAL", 85, 40);
        mylognormal=rand("LOGNORMAL", 1, 0.1);
        mybeta=rand("BETA", 1,99);
        mybinary=rand("BINOM", 0.95, 1);
        mybinomial=rand("BINOM", 0.95, 2000);
        myexponential=rand("EXPONENTIAL", 4.2);
        mygamma=rand("GAMMA", 2, 2);
        mygeometric=rand("GEOMETRIC", 0.10);
        mynegbinomial=rand("NEGBINOMIAL", 0.97, 1000);
        mypoisson=rand("POISSON", 10);
        output;
    end;

PROC PRINT data=rand(obs=20);

PROC UNIVARIATE data=rand;
    var mynormal mylognormal mybeta mybinary mybinomial myexponential mygamma mygeometric
        mynegbinomial mypoisson;
    histogram;

    *can change around the parameters;
*-------------------------------------------------------------------------------------------------------------------------------;

*Examples of distributions in action;

    *Datasets;
DATA Fish; set sashelp.Fish;
PROC PRINT data=Fish(obs=25);

DATA Baseball; set sashelp.Baseball;
PROC PRINT data=Baseball(obs=25);


    *Normal;
PROC UNIVARIATE data=Fish;
```

/home/markwilliamson20/my_courses/markwilliamson0/MW_2021_Work/Module Examples/Distributions Examples.sas UTF-8

Messages | User: markwilliamson20

Code available at: https://med.und.edu/daccota/_files/docs/berdc_docs/distributions_sas_code.txt

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R examples

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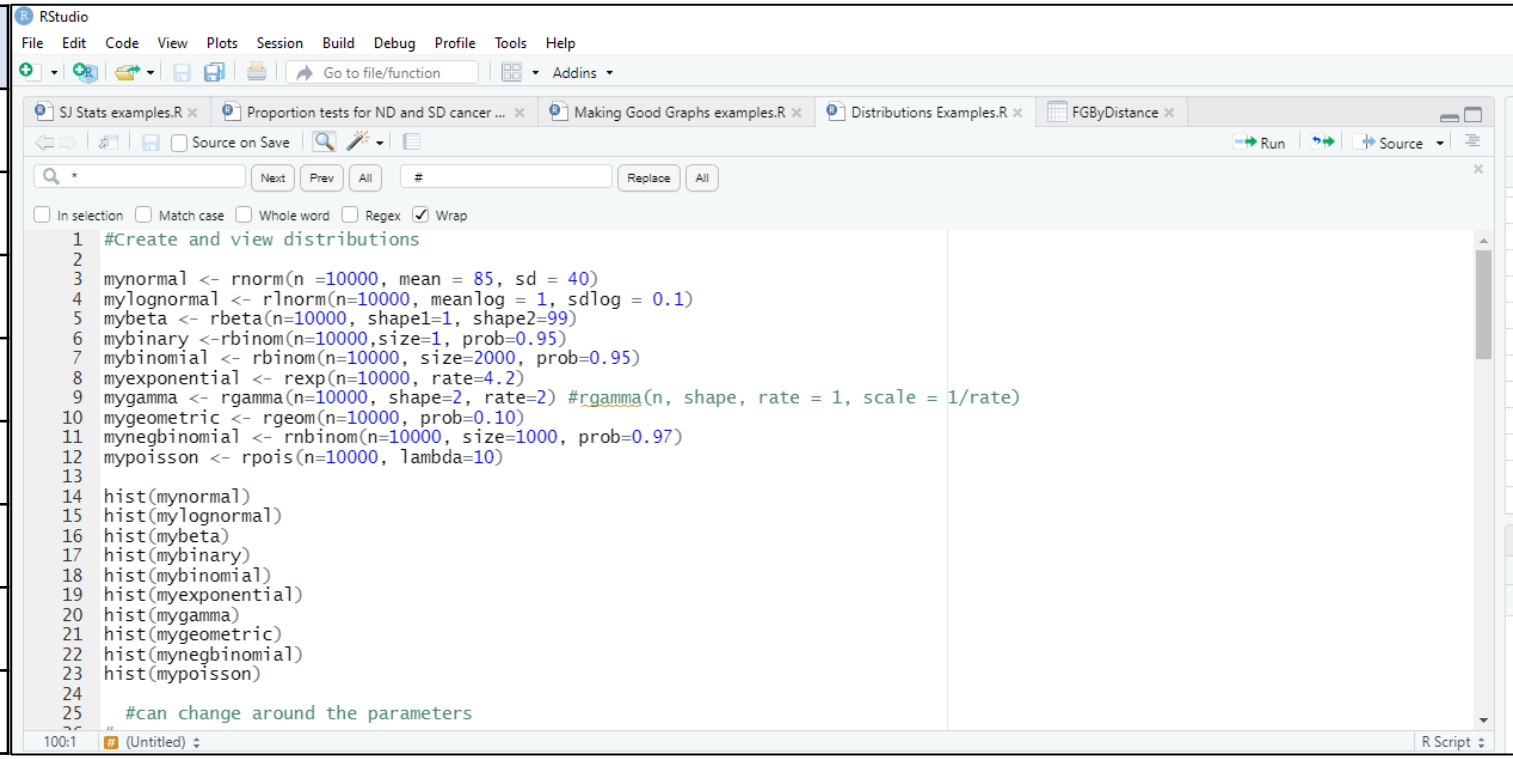
Family	Default Link Function
binomial	(link = "logit")
gaussian	(link = "identity")
Gamma	(link = "inverse")
inverse.gaussian	(link = "1/mu^2")
poisson	(link = "log")
quasi	(link = "identity", variance = "constant")
quasibinomial	(link = "logit")
quasipoisson	(link = "logit")



Code available at: https://med.und.edu/daccota/_files/docs/berdc_docs/distributions_r_code.txt

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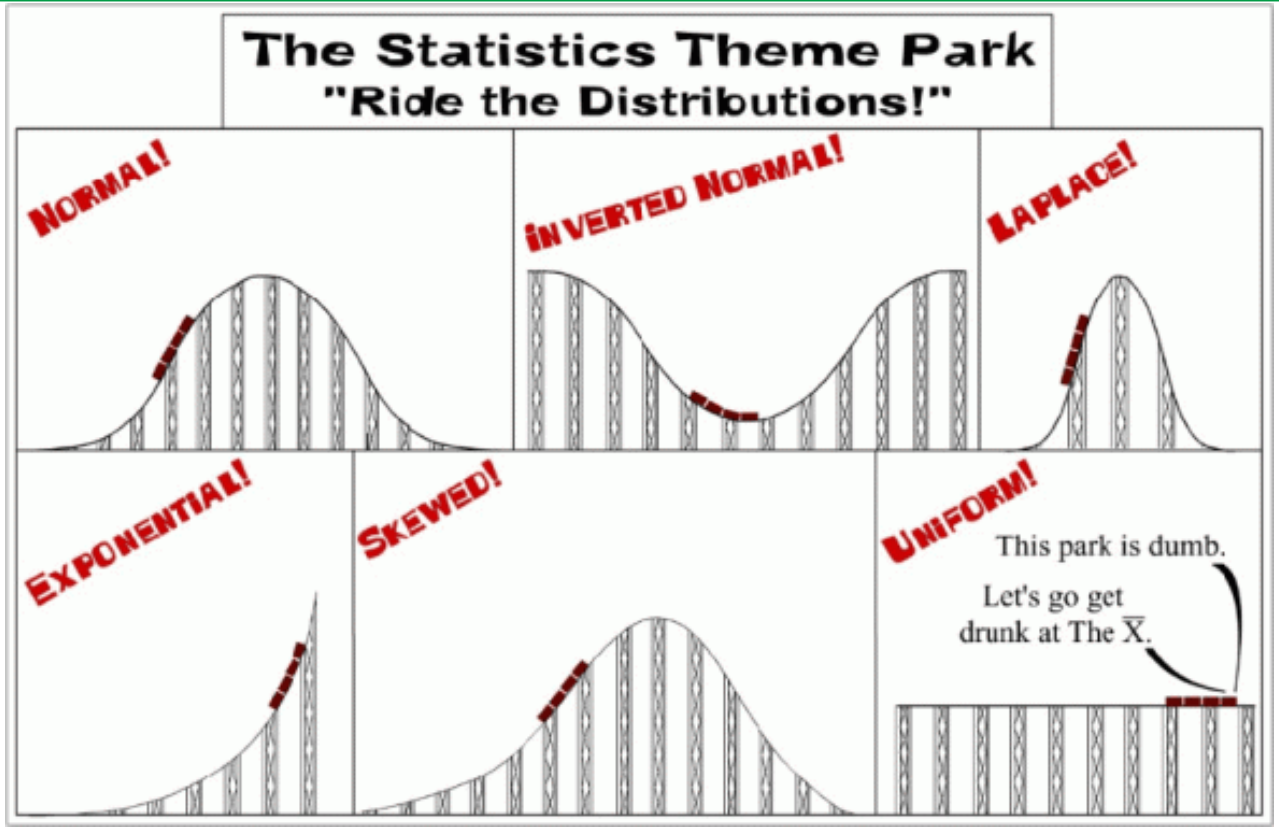
Wrap-up



Please take the post-test and survey:

Post-test: https://und.qualtrics.com/jfe/form/SV_5swKhmpU4tO7hau

Survey: https://und.qualtrics.com/jfe/form/SV_5o370oBxAOTtMr4

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References

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Images:

- https://www.wolfram.com/mathematica/newin6/content/SymbolicStatisticalComputing/BuiltinStatisticalDistributions.html
- https://pierpaolo28.github.io/blog/blog19/
- https://courses.lumenlearning.com/wmopen-concepts-statistics/chapter/continuous-probability-distribution-2-of-2/
- https://www.spss-tutorials.com/sampling-distribution-what-is-it/
- https://medium.com/analytics-vidhya/probability-distributions-444e7babf2e1
- https://www.kdnuggets.com/2020/02/probability-distributions-data-science.html
- https://www-users.cse.umn.edu/~dodso013/fm503/0910/lectures/fall2.pdf
- https://tinyheero.github.io/2016/03/17/prob-distr.html
- https://en.wikipedia.org/wiki/List_of_probability_distributions
- https://loonylabs.files.wordpress.com/2019/10/distribution-park-joke.gif?w=590

Materials:

- https://365datascience.com/tutorials/statistics-tutorials/distribution-in-statistics/
- https://www.itl.nist.gov/div898/handbook/eda/section3/eda361.htm
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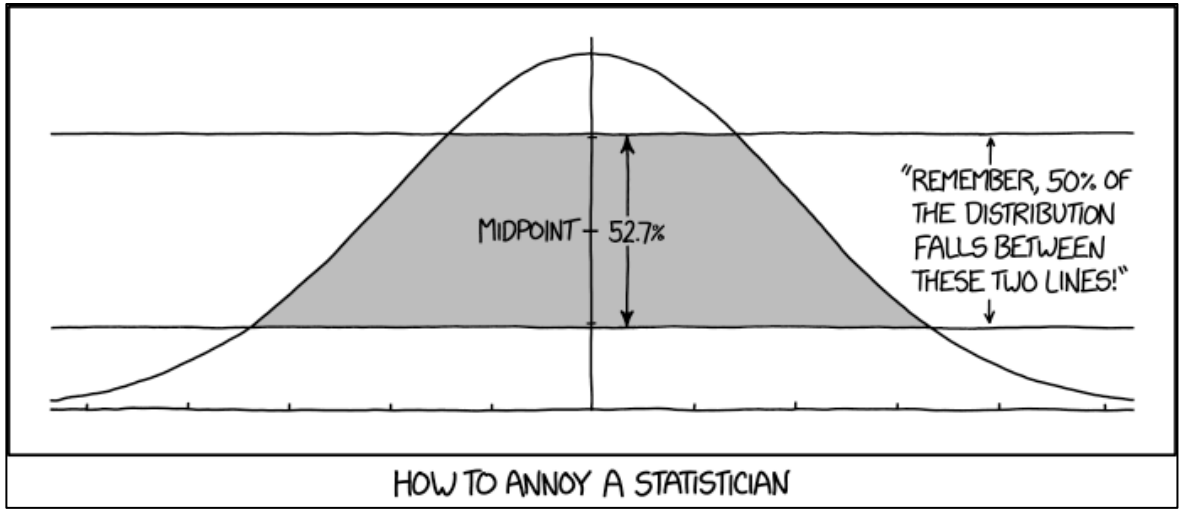

https://xkcd.com/2118/

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