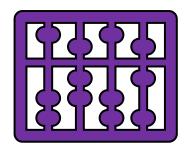
Resampling Magic

NORTH DAKOTA.



BERDC Special Topics Talk 18



DaCCoTA

DAKOTA COMMUNITY COLLABORATIVE ON TRANSLATIONAL ACTIVITY

Dr. Mark Williamson

Biostatistics, Epidemiology, and Research Design Core



Introduction



Goal: Spell out the benefits and methods of resampling

- What are the major types?
- How does resampling work?
- Worked examples









Solve Every Statistics Problem with One Weird Trick [1]

- Resampling and Visualization
- ✓ Monte Carlo
- Confidence Intervals
- Significance Testing
- **✓** Error Detection
- *And Beyond









* Resample Definition:

"Creation of new samples based on one observed sample."
-Wikipedia [2]

"Series of techniques used in statistics to gather more information about a sample. This can include retaking a sample or estimating its accuracy."

-Indeed [3]





The Types [4-7]

Jackknife

Randomization

Monte Carlo

K-fold validation

Bootstrap

Permutation

Cross-Validation

Normal Resampling







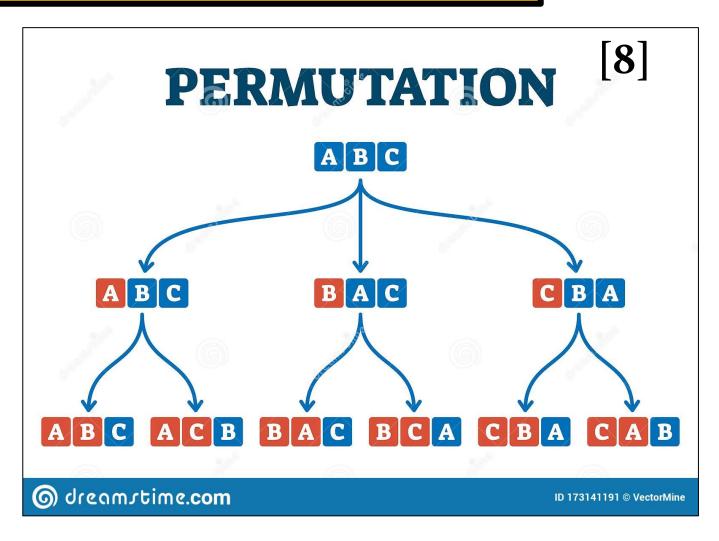
- Null Hypothesis significance testing (permutation)
- *Accuracy, confidence intervals (bootstrap)
- Sampling distribution of test statistic (bootstrap)
- ✓ Validating a predictive model (jackknife, Monte Carlo, etc.)







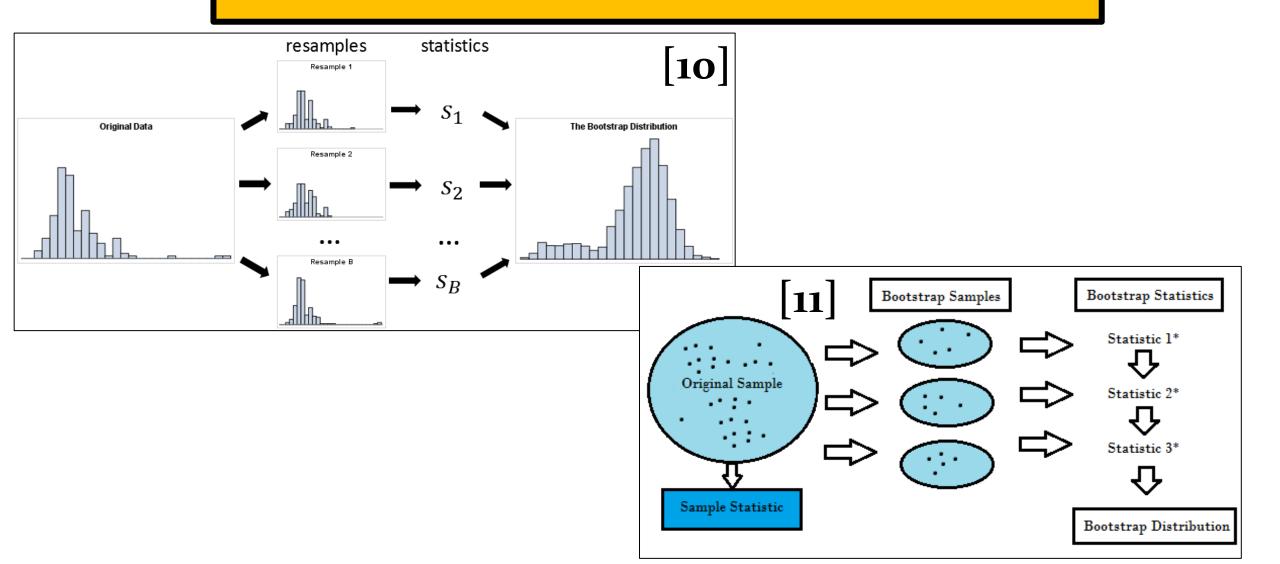
- Mostly done by computers
- ♠ R, SAS, Python, etc.
- Need dataset and objective







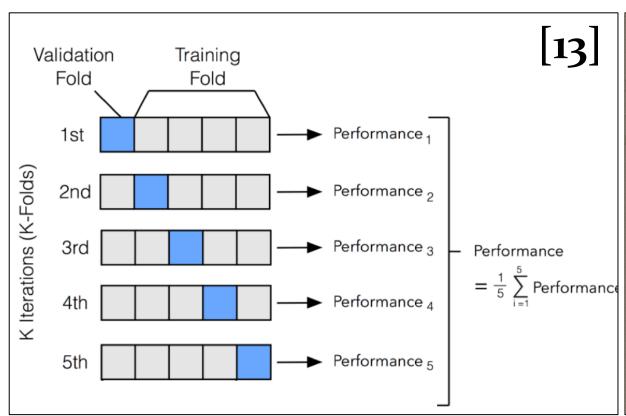


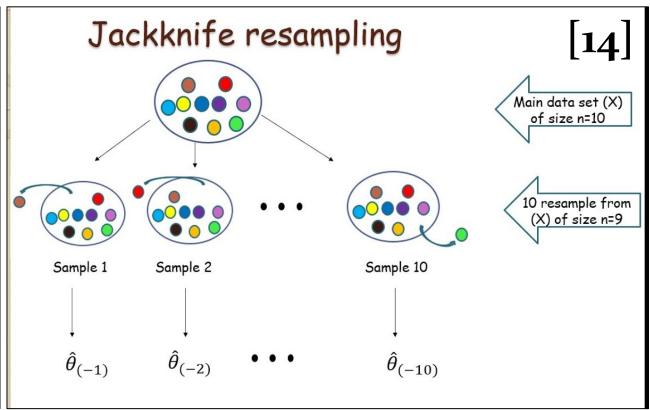




The How [12]



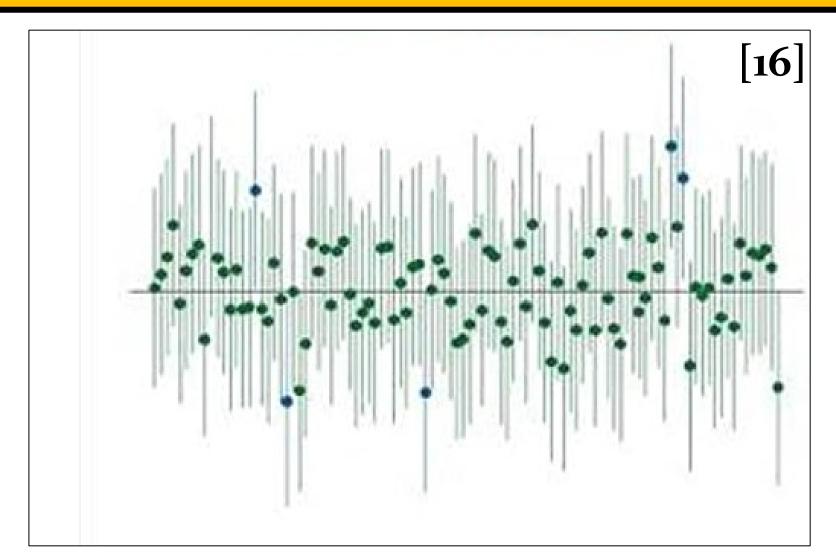








The How [15]









Dataset

Magic the Gather Playing card data [17]

> head(magic)

Name	Colors	Set Code	Mana Cost	Power	Toughness	Types
Ancestor's Chosen	W	10	7	4	4	Creature
Angel of Mercy	W	10	5	3	3	Creature
Angelic Blessing	W	10	3	NA	NA	Sorcery
Angelic Wall	W	10	2	0	4	Creature
Aven Cloudchaser	W	10	4	2	2	Creature
Benalish Knight	W	10	3	2	2	Creature
•••	•••	•••				• • •

```
#Resampling Magic R-code:
#References:
      https://rcompanion.org/handbook/K_01.html
      https://www.statology.org/bootstrap-standard-error-in-r/
     https://stackoverflow.com/questions/58393608/bootstrapped-correlation-in-r
     https://www.statology.org/k-fold-cross-validation-in-r/
      #https://search.r-project.org/CRAN/refmans/pls/html/jack.test.html
#Get data and packages
library(lmPerm)
library(dplyr)
library(ggplot2)
library(boot)
library(caret)
library(pls)
  #magic cards:
setwd("C:/Users/Mark.Williamson.2/OneDrive - North Dakota University System/Desktop/Williamson Data/Magic")
magic<-read.csv("cards_condensed_10E_clean.csv")</pre>
head(magic)
boxplot(magic$convertedManaCost~magic$colors, col=c("grey", "green", "red", "blue", "white"))
boxplot(magic$convertedManaCost[magic$types=="Creature"]~magic$colors[magic$types=="Creature"],
        col=c("grey", "green", "red", "blue", "white"))
boxplot(magic$toughness[magic$types=="Creature"]~magic$colors[magic$types=="Creature"],
        col=c("grey", "green", "red", "blue", "white"))
boxplot(magic$power[magic$types=="Creature"]~magic$colors[magic$types=="Creature"],
        col=c("grey", "green", "red", "blue", "white"))
creatures <-magic[magic$types=="Creature",]
```

```
par(mfrow=c(2.3))
hist(creatures$convertedManaCost[creatures$colors=="R"], col="red", main="Mana Cost", xlab="Cost")
hist(creatures$convertedManaCost[creatures$colors=="U"], col="blue", main="", xlab="Cost")
hist(creatures$convertedManaCost[creatures$colors=="B"], col="grey", main="", xlab="Cost")
hist(creatures \ converted Mana Cost \ [creatures \ colors == "W"], col = "white", main = "", x \ lab = "Cost")
hist(creatures \ converted Mana Cost \ [creatures \ colors == "G"], col = "green", main = "", x \ lab = "Cost")
par(mfrow=c(1.1))
par(mfrow=c(2.3))
hist(creatures power[creatures colors=="R"], col="red", main="Power", xlab="Power")
hist(creatures power[creatures colors=="U"], col="blue", main="", xlab="Power")
hist(creatures power[creatures colors=="B"], col="grey", main="", xlab="Power")
hist(creatures$power[creatures$colors=="W"], col="white", main="", xlab="Power")
hist(creatures power creatures colors=="G"], col="green", main="", xlab="Power")
par(mfrow=c(1.1))
par(mfrow=c(2.3))
hist(creatures toughness [creatures colors == "R"], col="red", main="Toughness", xlab="Toughness")
hist(creatures $toughness [creatures $colors == "U"], col="blue", main="", xlab="Toughness")
hist(creatures$toughness[creatures$colors=="B"], col="grey", main="", xlab="Toughness")
hist(creatures$toughness[creatures$colors=="W"], col="white", main="", xlab="Toughness")
hist(creatures toughness [creatures colors=="G"], col="green", main="", xlab="Toughness")
par(mfrow=c(1,1))
creatures %>%
  group_by(colors) %>%
  summarize(mean_Mana = mean(convertedManaCost),
             mean_power= mean(power).
             mean_toughness=mean(toughness))
```







Permutation Test

Is average power different between Green and White creatures?



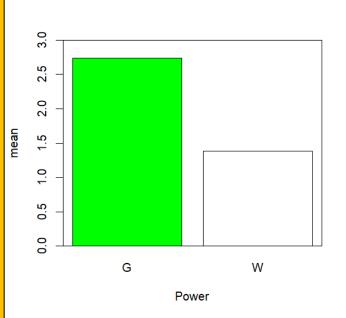


```
#Example 1: Permutation test
    #Question: Is average power different between Green and White creatures?
creatures_GW <-subset(creatures, colors == "G" | colors == "W")
head(creatures_GW)

glm1 <- glm(power ~ colors, data=creatures_GW, family="poisson")
summary(glm1)

lm1 <-lm(power ~ colors, data=creatures_GW)
summary(lm1)
anova(lm1)
barplot(c(2.74,1.38)~c("G","W"), xlab="Power", ylab="mean", col=c("Green", "White"), ylim=c(0,3))
box()

lmp1 <-lmp(power ~ colors, data=creatures_GW, perm="Prob", seqs=FALSE)
summary(lmp1)</pre>
```



```
> summary(glm1)
glm(formula = power ~ colors, family = "poisson", data = creatures_GW)
Deviance Residuals:
                    Median
-2.34247 -0.71120 0.08007 0.15248 2.57099
Coefficients:
           Estimate Std. Error z value Pr(>|z|)
                      0.09667 10.440 <2e-16 ***
(Intercept) 1.00927
colorsW
         -0.37328
                     0.15509 -2.407 0.0161 *
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
(Dispersion parameter for poisson family taken to be 1)
   Null deviance: 73.171 on 74 degrees of freedom
Residual deviance: 67.247 on 73 degrees of freedom
AIC: 262.53
Number of Fisher Scoring iterations: 5
```

anova(1mp1)

```
> summary(lm1)
Call:
lm(formula = power ~ colors, data = creatures_GW)
Residuals:
            10 Median
-2.7436 -0.8889 0.1111 0.2564 5.2564
Coefficients:
           Estimate Std. Error t value Pr(>|t|)
(Intercept) 2.7436
                       0.2479 11.066 <2e-16 ***
colorsW
            -0.8547
                       0.3578 -2.388 0.0195 *
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '1
Residual standard error: 1.548 on 73 degrees of freedom
Multiple R-squared: 0.07248, Adjusted R-squared: 0.05978
F-statistic: 5.705 on 1 and 73 DF, p-value: 0.01951
> anova(lm1)
Analysis of Variance Table
Response: power
         Df Sum Sq Mean Sq F value Pr(>F)
colors 1 13.675 13.6752 5.7048 0.01951 *
Residuals 73 174.991 2.3971
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
> summary(1mp1)
call:
lmp(formula = power ~ colors, data = creatures_GW, perm = "Prob",
   seqs = FALSE)
Residuals:
           1Q Median 3Q
-2.7436 -0.8889 0.1111 0.2564 5.2564
Coefficients:
       Estimate Iter Pr(Prob)
colors1 0.4274 2732 0.0355 *
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '1
Residual standard error: 1.548 on 73 degrees of freedom
Multiple R-Squared: 0.07248, Adjusted R-squared: 0.05978
F-statistic: 5.705 on 1 and 73 DF, p-value: 0.01951
> anova(lmp1)
Analysis of Variance Table
Response: power
        Df R Sum Sq R Mean Sq Iter Pr(Prob)
Residuals 73 174.991 2.3971
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '1
```







Permutation Test

Is average toughness different between Green and Black creatures?





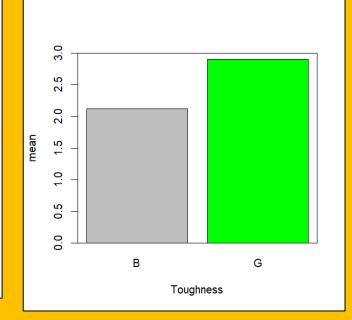
```
#Question: Is average toughness different between Black and Green creatures?
creatures_BG <-subset(creatures, colors == "B" | colors == "G")
head(creatures_BG)

glm2 <- glm(toughness ~ colors, data=creatures_BG, family="poisson")
summary(glm2)

lm2 <-lm(toughness ~ colors, data=creatures_BG)
summary(lm2)
anova(lm2)

barplot(c(2.12,2.90)~c("B","G"), xlab="Toughness", ylab="mean", col=c("Grey", "Green"), ylim=c(0,3))
box()

lmp2 <-lmp(toughness ~ colors, data=creatures_BG, perm="Prob", seqs=FALSE)
summary(lmp2)
anova(lmo2)</pre>
```



```
> summary(g1m2)
glm(formula = toughness ~ colors, family = "poisson", data = creatures_BG)
Deviance Residuals:
    Min
              1Q Median
-1.29121 -0.85933 -0.08404 0.56760 2.63768
Coefficients:
           Estimate Std. Error z value Pr(>|z|)
(Intercept) 0.7520
                      0.1195 6.292 3.14e-10 ***
colorsG
             0.3118
                       0.1521 2.050 0.0403 *
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
(Dispersion parameter for poisson family taken to be 1)
    Null deviance: 67.728 on 71 degrees of freedom
Residual deviance: 63.438 on 70 degrees of freedom
AIC: 260.04
Number of Fisher Scoring iterations: 5
```

```
> summary(lm2)
lm(formula = toughness ~ colors, data = creatures_BG)
Residuals:
           1Q Median
-1.8974 -1.1212 -0.1212 0.8788 5.1026
Coefficients:
           Estimate Std. Error t value Pr(>|t|)
(Intercept) 2.1212
                      0.2815 7.534 1.31e-10 ***
             0.7762
                       0.3825 2.029 0.0462 *
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 1.617 on 70 degrees of freedom
Multiple R-squared: 0.05555, Adjusted R-squared: 0.04206
F-statistic: 4.117 on 1 and 70 DF, p-value: 0.04625
> anova(1m2)
Analysis of Variance Table
Response: toughness
         Df Sum Sq Mean Sq F value Pr(>F)
colors 1 10.77 10.7701 4.1174 0.04625 *
Residuals 70 183.10 2.6158
Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

```
> summary(1mp2)
lmp(formula = toughness ~ colors, data = creatures_BG, perm = "Prob",
   seqs = FALSE)
Residuals:
           1Q Median
-1.8974 -1.1212 -0.1212 0.8788 5.1026
Coefficients:
       Estimate Iter Pr(Prob)
colors1 -0.3881 3148 0.0308 *
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 1.617 on 70 degrees of freedom
Multiple R-Squared: 0.05555, Adjusted R-squared: 0.04206
F-statistic: 4.117 on 1 and 70 DF, p-value: 0.04625
> anova(1mp2)
Analysis of Variance Table
Response: toughness
         Df R Sum Sq R Mean Sq Iter Pr(Prob)
Residuals 70 183.10
                    2.6158
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '1
```



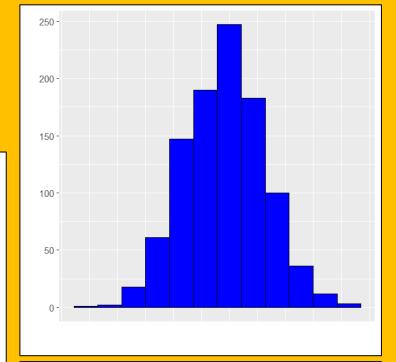


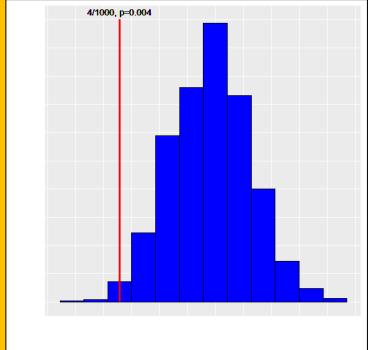


Permutation Test

Visualization

```
#visualization:
t_coeff<-unname(lm1$coefficients[2]) #estimate of difference between Green and White creature power
set.seed(10)
coeff_list <-c()
for (i in seq(1:1000)) {
  perm_colors<-sample(creatures_GW$colors,replace=FALSE)</pre>
  perm_power <-creatures_GW$power</pre>
  perm_df <-data.frame(colors=perm_colors, power=perm_power)</pre>
  plm <-lm(power ~ colors, data=perm_df)
  perm_coeff <-unname(plm$coefficients[2])</pre>
  coeff_list <<-c(coeff_list, perm_coeff)</pre>
coeff_df <-data.frame(coeff=coeff_list)</pre>
plot1 <-ggplot(data=coeff_df, aes(x=coeff))+
 geom_histogram(, bins=12, fill="blue", col="black")
plot1
t_coeff #actual coefficient we got
1000-sum(coeff_list>t_coeff)
plot1+
  geom_segment(aes(x=t_coeff, y=0, xend=t_coeff, yend=250), colour="red", linewidth=1)+
  geom_text(aes(label=c("4/1000, p=0.004")), x=t_coeff, y=257, size=3)
```











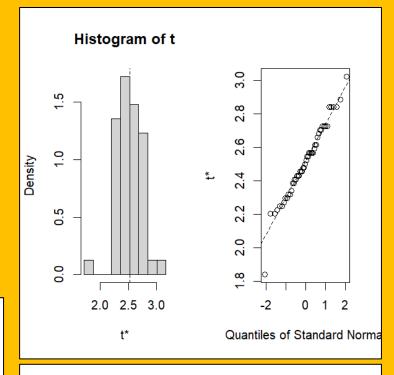
Bootstrapping

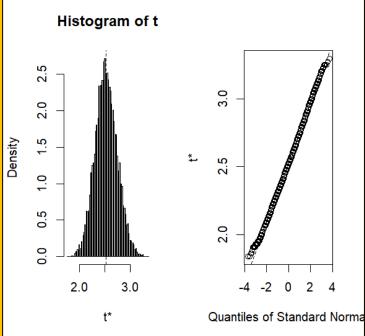
What is the bootstrap confidence in mean cost for Instants?



```
#Example 2: Bootstrapping
  #Question: What is the bootstrap confidence in mean cost for Instants?
head(magic[magic$types=="Instant",])
Instant_Cost <-magic$convertedManaCost[magic$types=="Instant"]</pre>
mean(Instant_Cost)
    #Bootstrap standard error
set.seed(10)
meanFunc <- function(x,i){mean(x[i])}</pre>
reps1 <-boot(Instant_Cost, meanFunc, 50)</pre>
reps2 <-boot(Instant_Cost, meanFunc, 10000)</pre>
    #Plot Bootstrap distribution
plot(reps1)
                                           > boot.ci(reps1, type="bca")
plot(reps2)
                                           BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
                                           Based on 50 bootstrap replicates
    #Calculate confidence intervals
boot.ci(reps1, type="bca")
                                           CALL:
                                           boot.ci(boot.out = reps1, type = "bca")
boot.ci(reps2, type="bca")
                                           Intervals:
                                           Level
                                                       BCa
                                           95% (2.071, 3.021)
                                           Calculations and Intervals on Original Scale
                                           Some BCa intervals may be unstable
                                           > boot.ci(reps2, type="bca")
                                           BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
                                           Based on 10000 bootstrap replicates
                                           CALL:
                                           boot.ci(boot.out = reps2, type = "bca")
                                           Intervals:
                                                       BCa
                                           Level
                                                (2.114, 2.977)
```

Calculations and Intervals on Original Scale









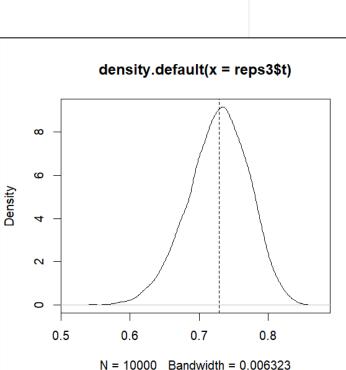


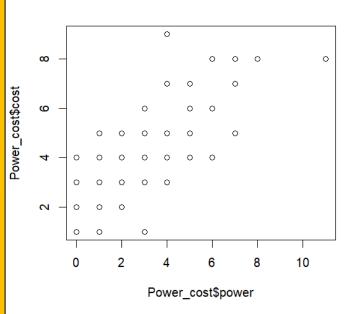
Bootstrapping

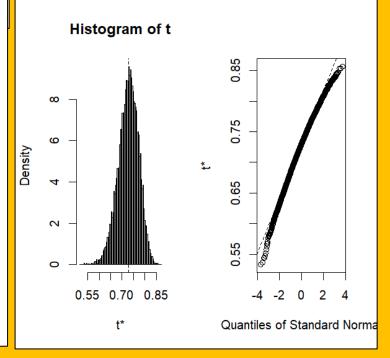
How confident can we be in a correlation between Mana cost and Creature power?



```
#Question: How confident can we be in a correlation between Mana cost and Creature power?
Power_cost <-data.frame(power=creatures$power, cost=creatures$convertedManaCost)
head(Power_cost)
cor1<-cor(Power_cost$power, Power_cost$cost, method="spearman")</pre>
cor1
plot(Power_cost$power, Power_cost$cost)
corFunc <- function(data,i){cor(data[i,"power"], data[i,"cost"], method="spearman")}</pre>
reps3 <-boot(Power_cost, corFunc, 10000)</pre>
reps3
plot(reps3)
boot.ci(reps3, type="bca")
  #density plot
```







boot.ci(reps3, type="bca")

BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS Based on 10000 bootstrap replicates

CALL:

boot.ci(boot.out = reps3, type = "bca")

Intervals:

Level BCa

plot(density(reps3\$t))

abline(v=cor1, lty="dashed")

(0.6252, 0.8024)

Calculations and Intervals on Original Scale





Examples

K-fold validation

How accurate is using power and toughness to predict Mana cost for creatures, using K-fold validation?



```
#Example 3: K-fold validation/ Jackknife
 #Question: How accurate is using power and toughness to predict Mana cost for creatures, using K-fold?
lm3 <-lm(convertedManaCost ~ power * toughness. data=creatures)</pre>
summary(1m3)
ctrl <- trainControl(method="cv", number =5)
modelA <-train(convertedManaCost ~ power, data=creatures, method="lm", trControl=ctrl)
modelB <-train(convertedManaCost ~ toughness, data=creatures, method="lm", trControl=ctrl)
modelC <-train(convertedManaCost ~ power + toughness, data=creatures, method="lm", trControl=ctrl)
modelD <-train(convertedManaCost ~ power * toughness, data=creatures, method="lm", trControl=ctrl)
   #RSME=root mean squared error: lower the better:
   #Rsquared=correlation: the higher the better:
   #MAE=mean absolute error: lower the better:
modelA$results; modelB$results; modelC$results; modelD$results
  #Model 4 had lowest RSME, highest Rsquared, and lowest MAE
modelD$finalModel
```

```
> summary(1m3)
                                                                         intercept
Call:
lm(formula = convertedManaCost ~ power * toughness, data = creatures)
                                                                         intercept
Residuals:
                                                                         intercept
   Min
            10 Median
                            3Q
-3.1036 -0.5313 -0.0468 0.4688 3.4379
                                                                         intercept
Coefficients:
               Estimate Std. Error t value Pr(>|t|)
(Intercept)
                0.56212
                           0.21331
                           0.09178
                0.72678
power
                           0.08303
                0.63933
                                    7.700 1.11e-12 ***
touahness
                                                                      Call:
power:toughness -0.06187
                           0.01446 -4.279 3.14e-05 ***
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '1
                                                                      Coefficients:
Residual standard error: 0.9487 on 168 degrees of freedom
Multiple R-squared: 0.6911,
                               Adjusted R-squared: 0.6856
F-statistic: 125.3 on 3 and 168 DF, p-value: < 2.2e-16
```

```
> modelA$results: modelB$results: modelC$results: modelD$results
                RMSE Rsquared
                                            RMSESD RsquaredSD
                                     MAE
                                                                   MAESD
      TRUE 1.100968 0.6108597 0.8388541 0.1124086 0.05188265 0.09605917
                RMSE Rsquared
                                            RMSESD RsquaredSD
                                     MAE
                                                                   MAESD
      TRUE 1.112877 0.5833777 0.8348641 0.0970927 0.04482937 0.06114345
                RMSE Rsquared
                                     MAE
                                             RMSESD RsquaredSD
                                                                    MAESD
       TRUE 1.005161 0.6442761 0.7370279 0.07047771 0.09868055 0.04877122
                 RMSE Rsquared
                                      MAE
                                              RMSESD RsquaredSD
                                                                     MAESD
      TRUE 0.9545428 0.6822408 0.6921889 0.09801078 0.08017481 0.09421169
   #Model 4 had lowest RSME, highest Rsquared, and lowest MAE
> modelD$finalModel
lm(formula = .outcome \sim ., data = dat)
      (Intercept)
                                              toughness
                                                          `power:toughness`
                               power
          0.56212
                             0.72678
                                                0.63933
                                                                  -0.06187
```





Examples

K-fold validation

How accurate is using power and toughness to predict Mana cost for creatures, using jackknife validation?



```
#Question: what about jackknife?
jlm <-pcr(convertedManaCost ~ power * toughness, data=creatures, validation="L00", jackknife=TRUE)
jack.test(jlm)
```



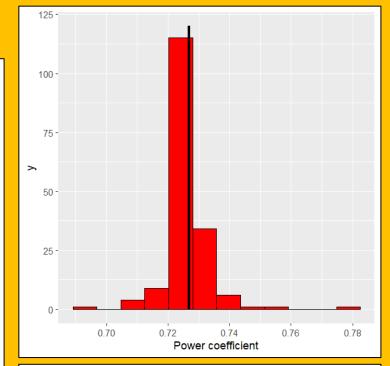


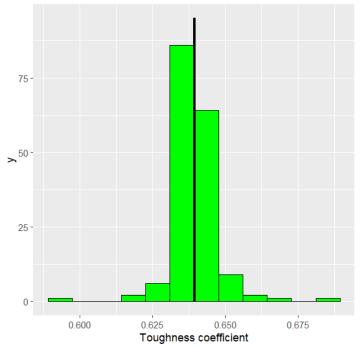


K-fold validation

Visualization (jackknife)

```
#visualization example:
lm3$coefficients
power_c <-unname(lm3$coefficients[2])
tough_c <-unname(lm3$coefficients[3])
lc<-length(creatures$convertedManaCost)</pre>
power_coeff_list <-c()
tough_coeff_list <-c()
for (i in seq(1:lc)){
  new_df <-creatures[-c(i), ]</pre>
  lm_loo <-lm(convertedManaCost ~ power * toughness, data=new_df)</pre>
  power_coeff <-unname(lm_loo$coefficients[2])</pre>
  tough_coeff <-unname(lm_loo$coefficients[3])</pre>
  power_coeff_list <<-c(power_coeff_list, power_coeff)</pre>
  tough_coeff_list <<-c(tough_coeff_list, tough_coeff)</pre>
power_jack_df <-data.frame(coeff=power_coeff_list)
tough_jack_df <-data.frame(coeff=tough_coeff_list)
plot2 <-ggplot(data=power_jack_df, aes(x=coeff))+
  geom_histogram(bins=12, fill="red", col="black")+
  geom_segment(aes(x=power_c, y=0, xend=power_c, yend=120), colour="black", linewidth=1.2)+
  labs(x="Power coefficient")
plot2
plot3 <-ggplot(data=tough_jack_df, aes(x=coeff))+
  geom_histogram(bins=12, fill="green", col="black")+
  geom_segment(aes(x=tough_c, y=0, xend=tough_c, yend=95), colour="black", linewidth=1.2)+
labs(x="Toughness coefficient")
plot3
```











Pr	Presentation				
[1]	https://www.youtube.com/watch?v=BhY-un6JURA				
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